GEOMETRICAL PREPARATION OF BACHELORS - FUTURE TEACHERS OF MATHEMATICS

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Abstract

The urgency of the problem stated in the article is due to the fact that the state standards of general education define the subject results of geometric preparation of students, and the state standard of training bachelors - future mathematics teachers spells out the professional competence of the teacher without regard to the profile and there are no approximate educational programs. The primary task of pedagogical universities is the development of a program of disciplines in geometry that meet modern requirements for the training of future teachers. It is necessary to choose the optimal volume and content of geometric disciplines, the most effective and rational ways of teaching students.

The article presents one of the possible options for the content of the program of geometric disciplines, developed by the department of teaching methods of mathematics, computer science and natural science disciplines of the Mari State University for the direction of training 44.03.05 Pedagogical education, Training profiles: Mathematics and Physics, qualification - bachelor. The program is the basis of the geometric training of future teachers of mathematics and takes into account the many years of experience of teachers in the department.

During the development of the program, the following were taken into account: the limited time allowed for the study of this course; a large amount of independent work of students; combination of the presentation of theoretical questions and illustration of their application in various branches of mathematics, physics and other natural sciences; relationship with the school course geometry. The program involves the preparation of a bachelor - a teacher of mathematics and physics, who must master all the sections of geometry considered in this article and it is easy to design a school course of geometry depending on changes in school curricula and textbooks.

The program involves teaching students how to solve practical problems using geometric methods. We consider both the simplest concrete examples illustrating the use of geometric concepts for the study of real phenomena, and special tasks from the physics course.

Keywords: geometric training bachelors, geometry course, interdisciplinary communication geometry

1. INTRODUCTION

The end of the XX and the beginning of the XXI centuries are characterized by the increasing role of mathematics, the application of its methods in various fields of knowledge. L.D. Kudryavtsev in [4] notes: "The question of what and how to teach in mathematics is again being sharply discussed at the present time in connection with the increasing role of mathematical methods both in solving specific practical problems and in conducting various theoretical studies."

This also applies to one of the most important branches of mathematics - geometry.

For example, in recent decades it has become customary to use such a geometric apparatus as vector and tensor algebra, vector and tensor analysis in some branches of physics, theoretical continuum mechanics, electrodynamics, semiconductor physics, the theory of relativity, chemistry, crystallography, power engineering, radio electronics, aerohydromechanics. This is due both to the convenience and clarity of the mathematical formulations of the laws, as well as the objective properties of the phenomena studied.

The basis of mathematical (in particular, geometric) education is laid in the primary and secondary (complete) school on the basis of the Federal State Educational Standard of Secondary General Education, which includes the following sections of geometry: vectors and the method of coordinates, plane and space transformations, image methods, measurements of geometric quantities and others.

2. MATERIALS AND METHODS

The program of training future teachers of mathematics and physics should provide for a close link between university and school geometry courses, but it is precisely in this question that the main problem arises. As noted above, the state standards of general education define the subject results of geometric preparation of school students, and the state standard of training bachelors - future mathematics teachers spells out the professional competence of the teacher without regard to the profile and there are no approximate educational programs.

Therefore, the priority task of pedagogical universities, faculties and departments is the development of a program of geometric disciplines that meet the modern requirements of training future teachers of mathematics.

The purpose of our research is to determine the content filling of geometric disciplines.

3. RESULTS

3.1 Content of Geometric Education of Future Teachers of Mathematics

We consider this question on the example of the Mari State University. The program of discipline B.1.23 "Geometry", the direction of training 44.03.05 Pedagogical education and training profiles: "Mathematics and Physics"; qualification - bachelor. The program is designed for 1-5 semesters (five years of study). The discipline volume is 15 credits, 540 hours.

In the first semester, students study 5 chapters: elements of vector algebra in space; coordinate method on a plane; straight line on the plane; second order lines; transform plane.

In the second semester - 5 chapters: the method of coordinates in space; planes and lines in space; space conversion; second order surfaces; multidimensional spaces, quadratic forms and quadrics.

In the third semester - 3 chapters: projective geometry, projective space; basic facts of projective geometry; image methods.

In the fourth semester - 5 chapters: elements of topology; polyhedra in Euclidean space; differential geometry, lines in Euclidean space; surfaces in Euclidean space; internal surface geometry.

In the fifth semester - 5 chapters: a historical overview of the substantiation of geometry, elements of Lobachevsky's geometry; general questions of axiomatics, justification of Euclidean geometry; length, area, volume; geometric constructions of the plane; non-euclidean geometries.

Selection and presentation of the material for all the sections listed above is designed so that with natural changes in the content of school textbooks on geometry (as is currently observed), future teachers could navigate the new situation and quickly restructure. The presentation of the material itself should be both thorough and accessible and prepare the basis for the usual content of elementary geometry. In other words, starting the study of geometry in high school from the sections in the first semester, the content of which is essentially the repetition, expansion and deepening of the questions of the relevant sections of elementary geometry, and continuing to study various sections of higher geometry, including sections of modern geometry - differential geometry and topology, in a new way, from a scientific point of view, look again at the content of school geometry.

3.2 Intra-Subject Relations of Geometrical Sections and Inter disciplinary Connections of Geometry and Natural Sciences

Differential geometry and topology

Modern geometry (geometry of the twentieth century and the beginning of the twenty-first century) is differential geometry and topology, but other sections also play a fundamental role in many areas of mathematics and its applications.

In turn, the rapid development of natural science disciplines (theoretical physics, mechanics, and other sciences) led to an understanding of the fundamental nature of geometric representations.

The training program provides for the study of elements of topology and classical differential geometry "in the small", as well as individual issues related to the concept of a topological manifold.

In more detail, students learn about individual issues of general topology and the theory of topological varieties in the corresponding special course "Elements of topologies" in the 9th semester. The total complexity of the discipline is 108 hours. The curriculum developed by the department provides for the study of two main sections: "Elements of the general topology" and "Elements of the theory of topological manifolds". An extended version of this special course is presented in [1].

Let us give one of the examples illustrating a certain connection between various sections of school and university courses in geometry, modern scientific research, as well as a connection with natural science disciplines. In [2], conjugate networks on a two-dimensional surface in four-dimensional Euclidean space are investigated. A research method is proposed that is associated with the introduction of two indicatrices of surface curvature, where part of the reasoning is directly related to one of the topics of the presented program “Indicatrix of Dupin and adjoint nets on the surface” (section “Differential Geometry”). But the latter is connected with the study of second-order curves in the “Analytical Geometry” section, which is the basis for studying the topic “Ellipse” and “Methods of image” of school and university geometry courses, in particular, images in parallel projection of two perpendicular diameters of a circle. This question is also closely related to the exact theory of the motion of celestial bodies created by Kepler, which made it possible to take a fresh look at the conic section theory developed by ancient Greek geometers and which, for about two thousand years, did not find its application.

We note another aspect. The program provides for acquaintance of students-mathematicians with specific issues of the theory of networks on the surface. In mathematical analysis, they got acquainted with curvilinear coordinate systems: polar, cylindrical, spherical, and in this course: the network of lines of curvature, asymptotic and conjugate networks, semi-geodetic and geodesic networks (an example of the latter is the usual Cartesian coordinate system of school courses of geometry and algebra). But if the researcher uses a curvilinear coordinate system, then he must be sure of the correctness of using this network. And here the question arises about the correctness of the network locally and as a whole. This question is rather difficult and has been the subject of research by a number of authors: N.V. Efimov, B.E. Cantor, E.R. Rozendorn, I.B. Barsky et al. We note only one work by I. B. Barsky “Asymptotic networks on a two-dimensional surface in three-dimensional and four-dimensional Euclidean spaces [5, p. 25-31].

Elements of vector algebra and the method of coordinates

The questions considered in these sections of geometry serve as the basis for the further study of almost all sections of higher mathematics, many sections of natural science and technical disciplines, and basic ones for solving problems of the school geometry course.

The connection of the section "Method of coordinates" and the corresponding sections of physics is illustrated, for example, by the article of A.M.Baranov "Coordinates and reference systems in physics" [6, p. 6-9].

Transformations of the plane and space

Geometric transformations play an important role, both in the school course of geometry, and in the course of geometry of higher education. It is clear that this topic is far from fully described in the school course of geometry. The same applies to certain courses of higher mathematics. Students of mathematics pedagogical direction study this topic most fully, and at school more fully questions of geometric transformations can be considered as special courses (elective courses, etc.). The theory of geometric transformations played an important role in the formation of views on geometry - a group approach to geometry (Felix Klein's Erlanger

Program).

Group methods had a great influence on the development of other sections of higher geometry, other mathematical disciplines and the natural sciences.

The first applications of the concept of groups found in algebra and developed in the theory of E. Galois, which is well known to students of mathematics. Also, students are well aware of groups of motion, similarity, and affine transformations in geometry.

Projective geometry. Methods of image.

In geometry, group methods originated in the study of projective geometry. From time immemorial, artists sought to depict spatial figures on a plane as they are visible from one fixed point. At the same time, the images of these figures must obey certain laws.

Later, the concepts of infinitely remote points and lines (J. Desargues) and, ultimately, the concepts of perspective (central projection) and projective space were introduced.

We also note the enormous role played by drawings in teaching. Randomly carrying out the drawing, the teacher must take into account the relevant laws of design (most often, the laws of parallel design), ensure that the drawings are illustrative and help students understand and mentally present the corresponding geometric patterns. It should be remembered that as the geometric figure will be depicted on the board, it will also be depicted in the notebooks of school and university students. Here the use of incomplete images, which have a large supply of free parameters, plays an important role.

Right drawings serve as the basis for the development of students' spatial imagination, and the frequent use of wrong drawings leads to the fact that even a weak intuition of students will be directed in the wrong direction. It should be borne in mind that the student years of boys and girls are considered the most favorable for the development of imagination and memory. Students should also be well aware that the requirement of image fidelity (drawing) is not sufficient for a complete understanding of the figure and the corresponding geometric relationships. So, a cone can be designed in a triangle, a cylinder in a rectangle, a cube in a square, etc. All these images are correct, but not vivid. Note also that if the original drawing is not is correct (there are certain errors in the image), then it cannot be considered visual (not to mention the fact that we can get the wrong solution or cannot solve the problem at all).

For obvious reasons, important issues of projection theory are not included in the considered section of the “Geometry” discipline. Which are included in other disciplines (descriptive geometry, drawing, etc.).

The foundations of geometry. Non-Euclidean Geometries

This section of higher geometry, as a rule, is read (in a relatively full volume) to students - mathematicians of pedagogical universities.

Here we discuss the history of axiomatics from Euclid to Hilbert, various systems of axioms, and ultimately the student must understand what the axiomatic rationale for geometry and all of mathematics in general means.

From non-Euclidean geometry, the program provides for a more detailed study of the geometry of N.I. Lobachevsky.

It should be noted that often ideas enriching mathematics with new concepts and methods came from various sections of the natural sciences (for example, the concept of a vector). But with respect to non-Euclidean geometries, there is a feedback. These sections were created within mathematics under the influence of its internal constructions (for example, centuries-old attempts of mathematicians to prove the 5th postulate of Euclid). But these new sections of geometry formed the basis for the creation of corresponding sections of modern physics. For example, Lobachevsky’s geometry has found application in the special theory of relativity, and Riemannian geometry served as the foundation of the general Einstein theory of relativity. Just as the theory of convex figures, and later convex sets has found application in mathematical economics, theory of elasticity, and other applied fields of knowledge.

Let us make one more important remark: the program presented above stipulates that the whole basic course of geometry is completed with the study of the section “Foundations of geometry” (as well as the section “Structural geometry”). This is due to two reasons: to link the course of geometry with the course of methods of teaching mathematics and with the program of pedagogical practice of students of 3-5 courses.

4. DISCUSSIONS

An important component of teaching geometry in high school is the research activity of a student, which consists of the following areas: students work in scientific circles; term papers on mathematics and methods of teaching mathematics; reports at various scientific conferences levels; publication of scientific papers; implementation of final qualifying work, etc.

Without going into details, we will list only some of the topics of recent student works in 2018 in the field of geometry, whose leaders were the authors of the article: "Some questions about the axiomatics of the school geometry course"; "A topological approach to the study of some topics of the school geometry course"; "Jordan's theorem and some of its applications"; "Methods of teaching problem solving on the topic:" Polygons "of a school geometry course".

Geometrical education of students at the university is completed by repetition and systematization of knowledge on individual issues of the “Program”, which are submitted for final certification in mathematics of graduate students of pedagogical specialties of universities (state exam). A study guide [3] has been published to help graduate students.

5. CONCLUSION

Years of experience in teaching geometry, continuous interaction with the leaders of educational institutions and teachers of mathematics allows us to conclude that the level of geometric training of students on the basis of the developed program is quite high. At the same time, we consider it necessary to preserve that minimum of the volume and content of the sections and topics of this program presented above.

We also note that learning how to solve practical problems by geometric methods is not the task of the main course of geometry, it is the task of special courses (physics, etc.). But the program provides that the simplest concrete examples illustrating the use of geometric concepts and methods for studying real phenomena, as well as students' knowledge of certain issues of modern research in the field of geometry are very useful. As for the connection with school geometry, the relevant sections of the university program should be completed with applications in the school geometry course.

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