THE CONTROVERSIAL ISSUE OF RELATIVISTIC MASS AND ITS IMPACT BEYOND SCIENCE

Talal Al-Ameen

Professor of Mathematics and Mathematical Physics, Prince Mohammad Bin Fahd University (PMU), Saudi Arabia, talal_alameen@yahoo.co.uk, tameen@pmu.edu.sa

Abstract

In the relativistic mass formula, Albert Einstein shows that mass increases with velocity. This claim appears so counter-intuitive that it must be investigated. It further claims that at $v = c$ (where v is the object's speed, and c the speed of light) the object's relativistic mass (m) increases to infinity (∞). This work considers whether the scrutiny of mathematics and physics can justify this claim, using Einstein's Radiation Beam Box Thought Experiment. Scientifically, the aim of this work is to show that Einstein's most celebrated equation E (Energy) = mc^2 holds if, and only if, the photon is conceived to be mass-ive and furthermore that it is possible for mass-ive objects (such as an electron) to speed as fast as the photon. Educationally, the aim of this work is to show that the historical route to mass-ive photon may be an instrument to resolve the pedagogical challenges presented by relativistic mass. The work also briefly considers whether conventions, within a field of scientific investigation, can have unfortunate implication of how theory is formulated. Socially, the work discusses the impact of relativity on issues such as the existence of an absolute truth – as understood by philosophers such as Kant, prior to relativity.

Keywords: Relativistic mass, Einstein's Radiation Beam Box Thought Experiment, The natural philosophies of Kant.

1 INTRODUCTION

Einstein used thought experiments to conceptualize complex scientific ideas by imagining real-life scenarios. We are unaware of any theoretical work that has attempted to repeat Einstein's thought experiments. In our opinion this is needed in order to test whether Einstein's thought experiments can stand the test of time. This is possible by substituting variables that Einstein would not have known of at the time of his thought experiments with equivalent alternatives. In this work, we can substitute Einstein's light beam thought experiment by replacing the beam box for a laser beam. The results of this substitution does indeed support Einstein's claim that mass increases with velocity even though length contracts. However, confirmation of Einstein's view only occurs when light (photons) is being measured. If we substitute photons for electrons so that an electron beam was emitted, then Einstein's conclusion will not be confirmed. Similarly, the emission of a neutrino beam would produce similar results as an electron beam. Modernising Einstein's beam box thought experiment and releasing any particle (other than light) would fail to support Einstein.

2 RADIATION BEAM BOX (LASER GUN) THOUGHT EXPERIMENT

What follows presents what might be considered an updated version of Einstein's *Radiation Beam Box* experiment. In our illustration (Fig. below) the radiation beam box is replaced by a laser gun, and Einstein's radiation beam pulse is replaced by a laser pulse (Einstein A., 1905, 1906)

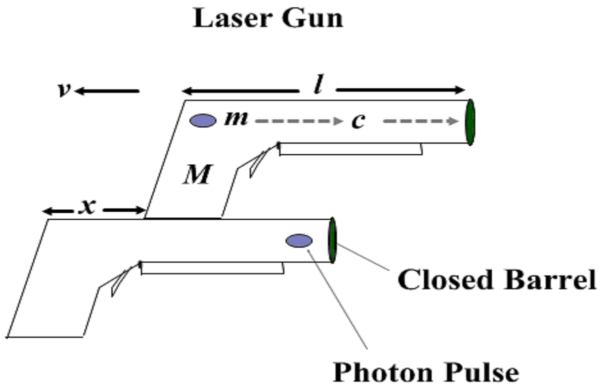

Fig. Showing a laser gun firing a laser pulse, however, the barrel of the gun is sealed by an impenetrable lead stopper. The top illustration of the laser gun depicts the laser pulse before the trigger of the laser gun is pulled. The length of the barrel is represented by *l*, the mass of the laser gun is represented by *M*, the '*mass'* of the laser pulse is *m*, the speed of the laser pulse (light) is *c*. When the gun is fired it recoils and its recoiling velocity is represented by *v*. When the gun recoils it moves back a distance, and this recoil distance is represented by *x*.

The reader should be aware that assigning mass to the laser pulse (*m*) is quite controversial as it assumes something that has not been established by science i.e. that light photons possess mass. The issue of the mass-ive photon presents a notorious dilemma for physicists. It is not by choice that we assign mass to the photon but as our illustration (Fig. above) follows Einstein's footsteps we are left with no other choice. It is, in our opinion, surprising that even though Einstein, who always maintained the photon to be mass-less, assigned *m* to his radiation beam (or pulse) he did not actually comment on the nature of this mass (relativistic or rest mass)! Sometimes supporters of Standard Special Relativity, denying that the photon is mass-ive, refer to the *m* of the radiation beam in terms of a *mass equivalence*, or *associated mass*, or even *mass in inertia*.

Consider the statement: "The photon does not carry mass but has the equivalence of mass!" Surely, it is tantamount to stating: "Jane is not pregnant but has a foetus in her uterus!" The only way to understand these statements is to admit that just as Jane carries a foetus the photon carries mass – no matter how infinitesimal.

If Einstein was of the opinion that the photon possessed mass (*m*) then it would be understandable why he assigned *m* to his radiation pulse. If he thought that radiation was something other than light then he might be justified in assigning mass to it, however, experimental evidence indicates that radiation is made up of light photons and Einstein was perfectly aware of this. Einstein's ardent support of the view that the photon was mass-less is especially evident in his debates with de Broglie, who in contrast, was ardently of the view that the photon was a 'mass-ive' particle.

Einstein's use of *E/c* in his derivations to represent the momentum of the radiation pulse, emitted in his radiation box model, indicates beyond any doubt that what Einstein meant by a radiation pulse could be nothing other than photons, as the light momentum can only be expressed by *E/c*, where *E* is the energy of the photon and ***c*** is of course the speed of the photon.

Einstein's derivations, based on his thought experiment depicted in the Fig., can now be considered. It is essential to review Einstein's derivations in order to understand how he came up with the formula $E=mc^2$ and how this led to his views on relativistic mass.

3 EINSTEIN'S DERIVATION OF $E = mc^2$

The first thing to take account of when considering Einstein's derivations is the principle of the conservation of momentum in an isolated system. The momentum of the gun's recoil (Mv) must be equal to the momentum of the photons from the laser's pulse (E/c). This should be clear as the momentum of the gun's recoil (Fig. above) is caused by the momentum of the photons, as depicted by:

$$Mv = E/c$$

$$v = E/Mc$$

The photons from the laser's pulse take time l/c to reach the barrel stopper when the trigger is pulled. During this time the gun recoils a distance x, therefore:

$$x = vt = v\ (l/c)$$

Substituting $v = E/Mc$ gives:

$$x = (E/Mc)\ l/c = El/Mc^2$$

The centre of the gun's mass (Fig. above) moves to the left because of the recoil and the photons from the laser's pulse will move to the right, hence:

$$Mx = ml$$

This relationship between Mx and ml is correct if, and only if, mass is assigned to the photons. Einstein here decides to overlook his own stance on mass, apparently supporting de Broglie, and perhaps inadvertently breaks away from the established view in Maxwell's electromagnetism equation that the photon can have no mass. There can be no doubt that Einstein himself, whether intentionally or inadvertently, assigns mass (even if called *equivalent mass*) to the photon in this equation:

$$m = Mx/l$$

If we now substitute distance x, ($x = El/Mc^2$) the following is obtained:

$$m = (M/\ l)\ (El/Mc^2)$$

Thus, emerges the most celebrated equation in physics:

$$E = mc^2$$

It is necessary to clearly understand, from this equation, what is actually meant by mass (m). It is generally assumed in physics that this m refers to the mass of any object in the universe but it is clear by the premise in Einstein's radiation box thought experiment that the equation presents m only as the mass (m) of a photon pulse.

4 THE INVALIDTY of $E = mc^2$ FOR AN ELECTRON BEAM

If the laser pulse is replaced, in the above thought experiment, with an electron pulse all that Einstein's thought experiment achieves is to tell us that the mass of the electron pulse (m_e) will equal the mass of the electron pulse. Let us demonstrate starting with:

$$Mv = E/c$$

This equation will no longer apply as this holds only for the photon pulse. E/c will be replaced with the mass of the electron pulse multiplied by the velocity of the electron (v_e):

$$M_{gun}\ v_{gun} = m_e\ v_e$$

This makes the whole calculation impotent:

$$v_{gun} = m_e\ v_e/M_{gun}$$

$$x = v_{gun}\ t = v_{gun}\ (\ l\ /\ v_e\ )$$

$$x = (m_e\ v_e/M_{gun}\ )(l\ /\ v_e)$$

$$M_{gun}\ x = m_e\ l$$

$$m_e = M_{gun}\ x\ /\ l$$

$$m_e = (M_{gun}\ /\ l\ )\ (m_e\ l\ /\ M_{gun}\ )$$

$$m_e = m_e$$

This would mean that Einstein's most celebrated $E = mc^2$ equation is inapplicable for the electron pulse, which would seriously limit its generalisation. $E = mc^2$ only survives as an equation that has general application to physical events if, and only if, the photon is conceived to be mass-ive and furthermore that it is possible for mass-ive objects to attain velocities in the order of the speed of light (c).

If supporters of Standard Special Relativity were able to accept such premises then $E = mc^2$ could still be generally applied; however, Standard Special Relativity ensures the death of Einstein's own most celebrated achievement by refusing to permit the possibility for any mass-ive object to travel at the speed of light (c). The general application of the equation $E = mc^2$ is only possible within a new model of Relativity.

5 FROM $E = mc^2$ TO RELATIVISTIC MASS

So how did Einstein get from $E = mc^2$ to his formulation of relativistic mass i.e. that increase in speed entails an increase in mass? The following will attempt to show how Einstein managed to formulate this relationship ($m_r = m_0 /(1- (v^2/c^2))^{1/2}$) from $E = mc^2$.

Returning to Einstein's radiation box (our laser gun) thought experiment the recoil velocity of the laser gun is expressed as:

$$v = dx/dt$$

$$dx = vdt$$

The work and/or energy (E) performed on the laser gun by the photons of the laser pulse is:

$$E= Fx$$

$$dE = Fdx$$

Here F is the force that causes the recoil of the gun, and dE is the change in the gun's energy due to the change in the energy of the moving photon pulse. And eliminating dx and F we get:

$$dE = Fvdt \qquad \text{where } dx = vdt$$

$$dE = d(Mv)\, v \qquad \text{where } F = d\,(Mv) / dt$$

$$dE = (Mdv+vdM)v$$

$$dE = vMdv + v^2dM$$

Having found the dE for the gun's energy Einstein now needs to find the dE for the change in the photon pulses energy. He uses the following method. In order to comply with the last derivative Einstein's equation $E = mc^2$ needs to be differentiated with respect to time:

$$E = mc^2$$

$$dE/ dt = d\,(mc^2)/dt$$

$$dE = c^2dm$$

By using this method Einstein not only assigns mass to the photon, as already stated, but differentiates the mass of the photon with respect to time as well: dm/dt. Yet, the use of this differentiation assumes time can affect mass, which is as yet unsupported by evidence from physics.

The answer to whether mass changes with time is to simply assert that such a relationship has never been observed in nature, although, as shown above, it is implied by Standard Special Relativity. Yet, if the law of conservation of energy and mass is still valid, and orthodoxy insists it is, then the mass of the universe over time must be considered as having remained constant. Thus, differentiating $E = mc^2$ with respect to time is a violation of nature (as far as mass and energy are concerned).

Even if Einstein, like de Broglie, accepted the notion of the mass-ive photon, and that this mass can change with time, then there would still be other conceptual problems in Einstein's equation for relativistic mass. Consider the following (see the above equations):

$$c^2dm = Mvdv + v^2dM$$

$$\int c^2dm = \int Mvdv + \int v^2dM$$

In the equation $c^2dm = Mvdv + v^2dM$, it would be mathematically problematic to consider $dm = dM$ (or $\int dm = \int dM$), for to do so would lead to $m = M$, which is impossible as it would require the laser gun recoiling a distance equal to the distance travelled by the photon pulse i.e. the impossibility of $x = l$. The laser gun's mass cannot be considered equal to the same mass of the photon pulse (i.e. the assumed mass of the photon). Unfortunately, these problematic assumptions are demonstrated by Einstein when he substitutes m for M, implying that they are equal, hence presenting us with:

$$c^2dm = mvdv + v^2dm$$

Einstein rearranged this equation in the following manner:

$$c^2dm - v^2dm = mvdv$$

$$dm\,(c^2 - v^2) = mvdv$$

$$dm/m = vdv/\,(c^2 - v^2)$$

Integrating this equation for m (from m_0 to m) and for v (from 0 to v) assumes that the rest mass (m_0) for the photon pulse is not zero i.e. that the photon pulse at rest is mass-ive, which contradicts Einstein's view that the rest mass of the photon should be zero. Furthermore, it is meaningless to assume a rest mass for the photon because by its nature a photon is never at rest. These integrations will result in:

$$\int_{m_0}^{m} \frac{dm}{m} = \int_{0}^{v} \frac{v}{(c^2 - v^2)}\,dv$$

or $$m \text{ (or } m_r) = m_0 / (1 - (v^2/c^2))^{1/2}$$

Even with all these problems, dv in the above equations presents an even greater problem for Einstein. In the equation dv indicates a change in velocity with respect to time, which simply means *acceleration*, when Standard Special Relativity assumes zero acceleration. If this is right then how is it possible for the recoiling gun in the Fig. to experience a change in its recoiling velocity (v)? The gun does not accelerate as it recoils because the velocity is unchanging. Einstein, himself understood this in his *tick-tock* thought experiment where the velocity (v) of the *moving clock* was constant but in the above equations Einstein presents us with a mass that experiences changes in velocity expressed by dv and thereby *acceleration*.

If Einstein demands consistency from his Special Relativity, then just as he insists that velocity remains constant (i.e. zero acceleration) for length contraction and time dilation then he should have assumed zero acceleration when considering relativistic mass; unfortunately, Einstein by allowing dv brings acceleration into his model. This is a major contradiction that casts a huge question mark over his Special Relativity. If he is referring to the same system, which he definitely is, then we would expect either dv to remain at zero (non-acceleration) for length, time and mass or alternatively to allow non-zero acceleration for all three. Regrettably, Einstein seems selective in his use of mathematics: he maintains zero acceleration for length contraction and time dilation but allows non-zero accelerations ($dv \neq 0$) for mass.

Assuming now that $dv = 0$, which orthodoxy accepts, and for the sake of argument accept that $dm \neq 0$ (i.e. m can change with *time*!) we get:

$$\int_{m_0}^{m} \frac{dm}{m} = \int_{0}^{v} \frac{v}{(c^2 - v^2)}\,dv = 0$$

or

$m / m_0 = 1$ (i.e. $m = m_0$) because $\ln 1 = 0$.

Thus, there can be no relativistic mass for the *moving object*:

$$m \text{ (or } m_r) = m_0 \neq m_0 / (1 - (v^2/c^2))^{1/2}.$$

It is clear that both logic and mathematical scrutiny causes the following expression to be deemed invalid:

$$m_r = m_0 / (1 - (v^2/c^2))^{1/2} \quad \text{invalid.}$$

It will be noticed that this equation is the relativistic momentum four-vector equation expressed in terms of mass. If this expression is now to be considered invalid for mass then perhaps the basis of the four-vector itself, as conceived by Einstein et al, needs re-evaluation.

If Einstein's derivation assumes that the rest mass of the photon is zero the relativistic mass would be indeterminate for the photon because it will be necessary to get involved in dividing zero by zero 0/0. This is because the factor $(1- v^2/c^2)^{1/2}$ is zero for the photon as the velocity of the photon is none-other-than the speed of light ($v = c$).

For the sake of argument let us assign rest mass to the photon, as de Broglie always insisted upon, and in recent years reliable research has suggested that the photon may indeed possess mass. De Broglie anticipated the mass of the photon to be in the order of 10^{-53}kg, which is significantly close to the upper limit inferred through satellite measurements of planetary magnetic fields. One such measurement carried out by the Charge Composition Explorer spacecraft was used to derive a geomagnetic limit on the photon mass m from an analysis of satellite measurements of the Earth's magnetic field. The upper limit for the photon mass was set at $m \leq 8\times10^{-16}$ eV/c^2 = 1×10^{-48}g (10^{-51} kg very close to de Broglie's value 10^{-53} kg) - *Source*: E. Fischbach et al. 1994.

The implication of photon mass, however infinitesimally small its mass may be, questions the validity of Einstein's relativistic mass formula because dividing non-zero by zero would mean that the energy (E) of the photon will increase to infinity, which would violate Planck's equation for the finite energy of photons $E = \hbar f$ (where $\hbar$ is the Planck constant and f is the frequency of the photon).

Again, even if Einstein's $m_r = m_0 /(1- v^2/c^2)^{1/2}$ is nominally accepted as being valid then it is still unable to save itself from its own demise unless Einstein is willing to accept two bitter conditions: the first one is to postulate that the velocity of the photon is smaller than the speed of light ($v_{photon} < c$), and secondly to accept that the rest mass of the photon is greater than zero ($m_0>0$). This avoids the problem of m_r stretching to infinity (when $v = c$ and $m_0 > 0$), or m_r ending up as: $m_r = 0/0$ (indeterminate) (when $v = c$ and $m_0 = 0$).

If supporters of Standard Special Relativity admit these changes then devastating consequences would overtake their model because its cornerstone would immediately erode. The censorship Special Relativity places upon the speed of light (*v of the photon must equal c*) will be violated and the demand that the rest mass of the photon being set at zero will also be contravened.

In addition, if these two bitter conditions ($v_{photon}< c$ and photon's $m_0> 0$) were to be embraced by Standard Special Relativity then Maxwell's electromagnetic equations would also be violated and would need modification.

6 EINSTEIN'S APPROACH TO THEORISING

In 1905 and 1915 respectively, Albert Einstein presented his Special Relativity and then his General Relativity to the physics community which, in the main, became quickly accepted as the standard model in physics. For more than a century, doubters have openly aired their doubts about the reliability of Einstein's work but none of these critics have been able to provide the mathematical proof to actually justify their doubts. Science tends to disregard theories that make assumptions without providing mathematical proofs. This is because physics is just as much indebted to mathematics as it is to experimentation and it is mathematics that clearly distinguishes a scientific theory from a conjecture.

Yet, even though the basis of Einstein's model is founded upon irrefutable mathematical equations there are conundrums in Einstein's model that physicists need to address. For example, the standard model assigns an *equivalent* mass to the photon but claims the photon has no mass; it claims that the speed of light is a censor in the universe yet speeds exceeding that of light have been measured by scientists in laboratory experiments and by cosmologists studying objects (quasars) in space. Our New approach to Special Relativity (ready to be published) will provide an alternative mathematical model, supported by observations that avoid the peculiarities confronting the standard model. It will also be demonstrated that our mathematical approach is "simpler" than Einstein's – and simplicity was considered by Einstein to be an essential element in a scientific theory. In 1933 he said:

> "Our experience hitherto justifies us in trusting that nature is the realization of the simplest that is mathematically conceivable. I am convinced that purely mathematical construction enables us to find those concepts and those law-like connections between them that provide the key to the understanding of natural phenomena. Useful mathematical concepts may well be suggested by experience, but in no way can they be derived from it. Experience naturally remains the sole criterion of the usefulness of a mathematical construction for physics. But the actual creative principle lies in mathematics. Thus, in a certain sense, I take it to be true that pure thought can grasp the real, as the ancients had dreamed." (Howard 2004, section 3).

Einstein's frequently characterised scientific theories as the "free creations of the human spirit." (Howard, Don A. (2004), ibid, Section 2). Within the necessary constraints for building a scientific theory, particularly limits set by mathematics and experience of the physical world, Einstein believed that the imagination of an individual scientist should be set free to experiment with thoughts. Einstein accepted that it is the nature of science for theories to change over the course of time in order to incorporate new discoveries and to explain conundrums and other internal problems within a theory. Einstein himself introduced an innovation into his own model (the cosmological constant) to explain why his original equations predicted an expanding universe. Remarkably, when Edwin Hubble discovered that the universe was indeed expanding in 1931 Einstein was happy to abandon his innovation as he had forced it over his original equations. Again remarkably, with the surprising discovery in 1998 that the expansion of the universe is accelerating, Einstein's cosmological constant provided an explanation for why the acceleration was occurring - implying the possibility of a positive nonzero value for the cosmological constant.

All scientific theories give rise to philosophical questions and implications for pedagogy but having limited experience in these disciplines our reflections on these issues may well be less than authoritative.

7 IMPLICATIONS FOR PHILOSOPHY

The most successful epistemologies of Einstein's era were those based on Immanuel Kant who had built his discussion of arithmetic reasoning and truth with corresponding claims about Euclidean geometry. As Kant's epistemological approach relied heavily on the principles of Euclidean geometry any doubts concerning the reliability of Euclidean geometry would be devastating to Kant's approach. Kant held that it was possible to have synthetic *a priori* truths in which the basic propositions of geometry and physics could be known *a priori* (independent of experience). Even before Einstein presented his General Relativity in 1915, the works of nineteenth century mathematicians had shown that geometries other than Euclid's were logically possible. Einstein's relativity theory could be seen as the final blow to the Kantian approach: In his 1921 essay Geometry and Experience, Einstein said:

> "... an enigma presents itself which in all ages has agitated inquiring minds. How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality? Is human reason, then, without experience, merely by taking thought, able to fathom the properties of real things? In my opinion the answer to this question is, briefly, this: as far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality..." (Norton, 2013, citing Einstein, Geometry and Reason,1921).

This was a thinly veiled repudiation of Kant because Einstein's German readers, steeped in Kantian philosophy, would have realised that Einstein was insisting that mathematical propositions, which include those of geometry, can be synthetic (i.e. "refer to reality") or a prior (i.e. certain"), but not both. Finally, in 1924, Einstein was crystal clear, "Until some time ago, it could be regarded as possible that Kant's system of *a priori* concepts and norms really could withstand the test of time. This was defensible as long as the content of later science held to be confirmed) did not violate those norms. This case occurred indisputably only with the theory of relativity. However, if one does not want to assert that relativity theory goes against reason, one cannot retain the *a priori* concepts and norms of Kant's system." (Norton 2013, ibid).

It may be argued that mathematically the foundations of Einstein's relativity could support *a priori* postulations. However, Gödel's Incompleteness Theorems showed mathematical knowledge is real knowledge about *something*, even though that something is not dependent in any way on our physical universe or on physical evidence. **(**Gödel, K., 2005). We, again, follow Einstein in rejecting the *a priori* concepts and norms of Kant's system.

8 IMPLICATIONS FOR PEDAGOGY

Some educationalists, enthusiastic about Einstein's theories of Relativity, have proposed that if students are to acquire what is unanimously accepted as the best scientific understanding of the universe then it would be better that they learn the fundamental concepts of Einstein's physics before those of Newton's physics. It seems reasonable to assume that if young and malleable minds learn about relativity first they would not face any contradictions in their prior knowledge as is the case for most students who encounter Newton first and then Einstein. This perspective assumes that if students grasped the concepts of relativity first, they would easily be able to learn that, according to Newton and for most of the time on Earth, we can treat time as being absolute, space as being flat, and gravity as a force field coming out of the Earth. This is because our perception and experience of the world coincides more with Newton's physics than with Einstein's. But Einstein's mathematics is more challenging so why expose students to that which is more difficult rather than scaffolding their learning in stages from Newton first and then to Einstein?

The Einstein-first approach, as it is called, argues that physics can be taught from an observational point of view, without recourse to complex abstract mathematics (Kaur et al, 2017). Also, in many respects Einstein's mathematics is no more complicated than those of Newtonian physics however we do not believe that the main difficulty for students learning about relativity derives from complex mathematical equations. The main obstacle for students is, in our opinion, the fact that many conclusions resulting from relativity are simply counter-intuitive. There can be little doubt that younger students, especially, would be convinced by and be more open to the observable conclusions of Newton than the more counter-intuitive ones provided by relativity. The fact that we are now proposing an alternative perspective to Einstein's Special Relativity would present further challenges to younger minds attempting to simply understand the subject of physics. Even though our New Special Relativity can resolve some of the peculiarities inherent in the standard model we do not recommend the Einstein-first or Relativity-first approach although it may well be an experiment worth trying. If this became a general trend in pedagogy then we would recommend that young learners should preferably be introducing initially to General Relativity rather than Special Relativity – as General Relativity is less fraught with conundrums.

9 CONCLUSIONS

This work has shown that Einstein’s most celebrated equation E (Energy) = mc^2 holds if, and only if, the photon is conceived to be mass-ive and furthermore that it is possible for mass-ive objects (such as an electron and neutrino) to speed as fast as the photon. Furthermore, this work argues that the historical route to mass-ive photon may be an instrument to resolve the pedagogical challenges presented by relativistic mass. Socially, the work confirms the impact of relativity on issues such as the existence of an absolute truth – as understood by philosophers such as Kant, prior to relativity. The work also confirms that conventions, within a field of scientific investigation, can have unfortunate implication of how theory is formulated.

10 ACKNOWLEDGEMENTS

I am grateful to Mr. Atila Orhan, for assistance with methodology; Prof. Imad Muhi and Prof. Michael Collins, Prince Mohammad Bin Fahd University (PMU), KSA, for comments and proofreading.

I am also grateful to Prof. M Elgeddawy for his unfailing support and assistance.

REFERENCE LIST

Einstein A., Annalenden Physik 17 {1905}: 891-921; Einstein A, Annalenden Physik 18 {1905}: pages 639-64; Einstein A, Annalenden Physik 20 {1906}: pages 627-633).

Gödel, K., Incompleteness: “The Proof and Paradox of Kurt Godel, Nature,” Vol: 435, Issue: 7038, 2005, Publisher: W. W. Norton & Company, Pages: 296.

Howard, Don A. (2004), "Einstein's Philosophy of Science", the Stanford Encyclopaedia of Philosophy, Edward N. Zalta (ed.). https://plato.stanford.edu/archives/spr2004/entries/einstein-philscience/

Kaur, T., Blair, D., Moschilla, J., Stannard, W., and Zadnik, M., Teaching Einsteinian Physics at Schools: Part 1, Models and Analogies for Relativity, (The University of Western Australia, 2017, https://arxiv.org/ftp/arxiv/papers/1704/1704.02058.pdf).

Norton, John. D, Einstein for Everyone, Philosophical Significance of the General Theory of Relativity. https://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/significance_GR_geometry/significance_GR_geometry.html#Kantian February 23, 2013.