Strategic Asset - Liability Optimization with Alternative Investments – for Life Insurance Companies

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Abstract. Standard portfolio theory deals with the optimal asset allocation for an asset-only investor. But especially institutional investors - like life insurance companies - have to match their strategic asset allocation decisions with their liabilities. This matching gets more important the longer the investment horizon. The following paper analyzes these inter-temporal relations between assets and liabilities for a Swiss life insurer. I include the typically in practice relevant asset classes and use state variables to predict returns, which have been identified in academic literature as powerful predictors. To capture the dynamics over the investment horizon I make use of a VAR(1) model and further calculate conditional moments. Therefore I take out predictability from movements so that the dynamics alter across the investment horizon. This makes it possible to anticipate variations in investment opportunities and to analyze portfolio implications from this perspective. The results from this analysis show that return predictability drives portfolio decisions and further how significant the consideration of liabilities and of horizon effects for a long-term asset-liability investor is.

1. Introduction

As institutional investors, life insurance companies allocate large amounts of assets to the financial markets. In their business, liabilities are a predetermined component of the balance sheet, which has to be considered while discussing strategic asset allocation decisions. To ensure an integrated portfolio optimization this balancing of risk and rewards on the asset side with the projected liability cash flows respectively the simultaneous view and integral management of assets and liabilities is critical. The central starting point of an integrated asset-liability management is the projection of future liability cash flows and based on that to ensure that asset allocation harmonizes with the dynamics of the liabilities.

The main goal of this paper is to analyze inter-temporal portfolio implications for a Swiss life insurance company. In this asset allocation analyzing task liabilities are replicated with well-known financial assets and considered as short position in the portfolio. Therefore the portfolio implications for an investor with liabilities will be different than for an asset-only investor, since assets that hedge against liability risk are valuable in this context. I analyze conditional dynamics (variances and covariances respectively correlations) over the time axis. That means that through the exclusion of the predictable components the conditional variances and covariances may not be constant over the horizon and therefore drive long-term portfolio choice.[1] I consider the portfolio choice of a Swiss life insurance company that can invest in the following asset classes: stocks Switzerland, stocks global, government bonds Switzerland, bonds global, private equity, hedge funds, commodities and listed real estate global. The remainder is invested in the benchmark asset, in this case the 3m CHF Libor rate.

The main part of life insurance related literature has spent effort on the development of models that perform stochastic modeling and simulations of the market development and the behavior of policyholders and further involved accounts. Most are focusing on fair valuation and contract design of unit-linked and participating insurance policies.[2] However, this paper has the focus on the asset side, but includes liabilities into the portfolio optimization problem. The market value of the assets depends on the initial endowment, on cash-in and outflows, which are expected to cancel each other out and on realized return. In practice the modeling of liabilities for a life insurance company is complex and depends on various factors. Simply put, returns on liabilities are interest expenses for a life insurer that have to be paid to the beneficiaries. They could be understood as a minimum required return on assets. Modeling is therefore straightforward and it can be expected that the inclusion of liabilities has an influence on asset allocation.[3] Furthermore, I assume a frictionless market; therefore I do not account for specific accounting treatment and optimization under this perspective, nor do I include taxation issues and transaction costs. Of course, these are essential factors to be considered for a real-life portfolio optimization. But, as I strive to develop generally applicable results about the interplay between assets and liabilities in a long-term asset allocation task and for simplicity I do not include them into the modeling. Nonetheless, by including them it must be expected that the portfolio implications will (slightly) differ compared to my solution. I am not aware of comparable studies analyzing the strategic portfolio implications for life insurance companies including liabilities.

Many papers analyze the impact of time-varying investment opportunities on the optimal investment strategy. The overwhelming part of these studies uses numerical methods. Further authors use analytical approaches. In this area of academic literature Campbell and Viceira [8], [9] introduce the "term-structure of the risk-return tradeoff", which demonstrates the implications of time variation on the risk-return tradeoff of bonds, stocks and a risk-free asset at different investment horizons. They argue that the mean-reversion characteristic of bond and stock returns generates inter-temporal hedge demands for these assets. Their approach can be extended to any number of asset classes and state variables. This paper follows their spirit. The estimated vector-autoregressive coefficients describe the return dynamics over the finite investment horizon. Uncertainty is modeled with stochastic process driving asset returns and state variables. My main concerns are the modeling of the liabilities and the estimation of the variance-covariance matrix for the time series analysis.
My contributions to the literature are the following. Firstly, I attempt to further bridge the gap between strategic portfolio optimization and asset-liability management. And secondly, I expand the portfolio choice on a wide range of asset classes, which are commonly used in practice. Thereby, I focus on

[1] If returns are not predictable variances and covariances would be constant over the horizon and long-term asset allocation would be equal to the one-period optimization task.
[2] See for example: [1], [2], [3], [4], [5], [6], [7].
[3] This paper only deals with horizon dependent effects between assets and liabilities. Differences between the two mentioned business lines are relevant for the modeling of risk aversion and therefore have no influence on the considered portfolios.

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alternative investments. My aim is to develop results that are applicable in practice for Swiss life insurance companies. I therefore model assets and liability returns somewhat different compared to previous studies.

This paper proceeds as follows. Section 2 is devoted to the description of the return dynamics of the VAR(1) model. Moreover, the portfolio problem is defined and the liability hedge portfolio (LHPF)[4] respectively the global minimum variance portfolio (GMV) and the tangency portfolio (for asset over liability returns) are derived. In the ensuing section data is specified and results of the estimation are presented. Moreover, implications over the investment horizon are analyzed. Section 4 discusses the portfolio implications and the role of alternatives investments. Therefore the LHPF is compared with the GMV portfolio. Moreover, implications from the tangency portfolio are discussed. Finally, results are critically evaluated. Section 5 gives the conclusions and highlights further strands of research.

2. Portfolio specification and horizon dynamics

This chapter specifies the VAR(1) model used to capture the return dynamics. Moreover, I define two portfolio problems. In the first case I assume that there is no risk-free asset available. For this case I derive the minimum-variance portfolio, which is denoted as LHPF for the asset-liability investor respectively as GMV portfolio for the asset-only investor. In the second case I assume that the investor has a risk-free asset available and further work with asset above liability returns (that is the net investment return after interest expenses on liabilities). For this case I calculate the tangency portfolio, which has no loading on the assumed risk-free asset.

2.1 Specifying the VAR(1) model

Asset and liability returns are assumed to follow a first-order vector autoregressive process. In general, a VAR(1) captures the linear interdependencies between time series, whereby the evolution of each of the variables is based on a constant, its own lags, lags of the other model variables and contemporaneous random shocks; it therefore fits coefficients that best describe the data at hand. In the following setting the dynamics of assets and liabilities are described by a VAR(1) process with monthly data. I specify

$$y_t = \begin{pmatrix} r_{rnom,t} \\ x_t \\ s_t \end{pmatrix},$$

where $r_{rnom,t}$ denotes the log return of the benchmark asset, x_t is a vector containing monthly log excess returns for each asset class and for liabilities with respect to the benchmark asset. The following asset classes are considered: stocks Switzerland (stch), stocks global (stgl), government bonds Switzerland (boch), bonds global (bogl), private equity (preq), hedge funds (hefu), commodities (comm) and listed real estate (reli). Liabilities (liab) refer to individual and collective life insurance – whereby it is not further distinguished between the two business lines. Finally, s_t is a vector of state variables, which is used to explain return dynamics. The following three state variables are considered in the analysis: dividend-price ratio (dpr), yield spread (ysp) and credit spread (csp).

The VAR(1) process can be specified as

$$y_{t+1} = \Phi_0 + \Phi_1 y_t + \varepsilon_{t+1} , \qquad [2.1]$$

[4] The GMV portfolio is the portfolio with the lowest variance for an asset-only investor. Throughout this paper I denote the portfolio with the lowest variance for the asset-liability investor as LHPF (see Chapter 2 for further details).

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where Φ_0 is a vector containing the intercepts; Φ_1 represents a square matrix of slope coefficients and ε_{t+1} is a white noise process with time invariant positive definite covariance $\Sigma\varepsilon\varepsilon$'.

However, the implied return dynamics let us gain information about how risk behaves across the investment horizon. Risk in this context refers to variances and covariances, considering that latter are for long-term investors at least as important as variances. The underlying concept was introduced by Campbell and Viceira [9] and is known as the term structure of risk. Input parameters are estimated from historical data, as described in the following chapter. The expressions derived below follow from the VAR(1) process defined in [2.1].

From forward recursion the following expression can be obtained

$$y_{t+n} = [\sum_{i=0}^{n-1} \Phi_1^i]\, \Phi_0 + \Phi_1^n\, y_t + \Sigma_{i=0}^{n-1} \Phi_1^i \varepsilon_{t+k-i} \qquad [2.2]$$

The conditional variance-covariance matrix can be described by

$$\begin{aligned} Var_t(y_{t+1} + \ldots + y_{t+n}) &= Var_t\, [[\Sigma_{i=0}^{n-1}(n-i)\, \Phi_1^i]\, \Phi_0 + [\Sigma_{j=1}^{n} \Phi_i^j]\, y_t + \sum_{q=1}^{n}[\, \Sigma_{p=o}^{n-q} \Phi_1^p \varepsilon_{t+q}]] \\ &= Var_t\, [\Sigma_{q=1}^{n}[\Sigma_{p=o}^{n-q} \Phi_1^p \varepsilon_{t+q}]] \qquad [2.3] \end{aligned}$$

This matrix contains conditional variances and covariances, which are used for the analysis of inter-temporal relations and therefore will have an impact on the mean-variance portfolio task.

2.2 Funding ratio perspective

The return from assets over liabilities is denoted as $r_{FR,t}$; $r_{A,t}$ is a vector aggregating the returns of each risky asset class and $r_{L,t}$ denotes the return of the liabilities. Liabilities are a short position in the insurer's portfolio and therefore $r_{A,t}$ is reduced by $r_{L,t}$ to obtain the funding ratio return $r_{FR,t}$. I work with logarithmic returns, as their continuously compounded return is symmetric. Since investors are concerned about gross returns whenever needed log returns are reversed. Furthermore, I work with nominal values for the modeling of assets and liabilities.

$$r_{FR,t} = r_{A,t} - r_{L,t} \qquad [2.4]$$

For simplicity, I further work with excess returns over the return of the benchmark asset. The 3m CHF LIBOR interest rate is referred to as benchmark asset. I denote $x_{A,t}$ as excess asset return and $x_{L,t}$ as excess liability return; $x_{A,t}$ and $x_{L,t}$ are captured by x_t,

$$x_t = \begin{pmatrix} r_{A,t} - \iota\, r_{rnom,t} \\ r_{L,t} - r_{rnom,t} \end{pmatrix} = \begin{pmatrix} x_{A,t} \\ x_{L,t} \end{pmatrix} \qquad [2.5]$$

whereas ι denotes a vector with ones.

2.3 Modeling of the predictable components of excess returns

Straightforwardly, the annualized expected excess return, $\mu_t^{(n)}$, and the annualized covariance matrix $\Sigma^{(n)}$ can be modeled,

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$$\mu_t^{(n)} = \frac{1}{n} E_t [x_{t+n}^{(n)}] = \begin{pmatrix} \mu_{A,t}^{(n)} \\ \mu_{L,t}^{(n)} \end{pmatrix}$$

$$\Sigma^{(n)} = \frac{1}{n} V_t [x_{t+n}^{(n)}] = \begin{pmatrix} \Sigma_{AA}^{(n)} & \sigma_{AL}^{(n)} \\ \sigma_{AL}^{(n)\prime} & \sigma_L^{2(n)} \end{pmatrix}$$

whereas $\Sigma_{AA,t}$ denotes the variance-covariance matrix of excess risky asset returns, σ_{AL} the covariance matrix of asset and liability returns and σ_L^2 the variance of liabilities.

2.4 Obtaining mean-variance frontier portfolio weights

The investor chooses a portfolio α_t of the risky assets and invests the remainder fraction of wealth $1-\alpha_t$ in the benchmark asset, $r_{rnom,t}$. The expected returns for assets and liabilities are set constant over time. The possibility of changes in volatility over the horizon is ignored. As in Campbell and Viceira, the following log-linear approximation can be derived

$$r_{A,t+1} = r_{rf,t+1} + \alpha_t' (r_{A,t+1} - r_{rnom,t+1}\,\iota) + \frac{1}{2}\alpha_t'\sigma_A^2 - \frac{1}{2}\alpha_t'\Sigma_{AA}\alpha_t \qquad [2.6]$$

Considering [2.4], the following equation can be obtained

$$\begin{aligned} r_{FR,t+1} &= \alpha_t' (r_{A,t+1} - r_{rnom,t+1}\,\iota) + \frac{1}{2}\alpha_t'\sigma_A^2 - \frac{1}{2}\alpha_t'\Sigma_{AA}\alpha_t - (r_{L,t+1} - r_{rnom,t+1}\,\iota) \\ &= \alpha_t' x_{A,t+1} + \frac{1}{2}\alpha_t'\sigma_A^2 - \frac{1}{2}\alpha_t'\Sigma_{AA}\alpha_t - x_{L,t+1} \end{aligned} \qquad [2.7]$$

Straightforward, the funding ratio log return is the log of a linear combination, which is not the same as a linear combination of logs. Therefore, the difference between the log funding ratio return and the linear combination of log returns is captured by $\frac{1}{2}\alpha_t'\sigma_A^2 - \frac{1}{2}\alpha_t'\Sigma_{AA}\alpha_t$ (correction of Jensen's inequality). Where σ_A^2 refers to the vector containing the diagonal elements of Σ_{AA}.

As mentioned, it is assumed that the investor rebalances to the initial weights at the end of each period. The aggregation of [2.7] to the n-period return with fixed, horizon-specific, portfolio weights $\alpha_t(n)$ leads to

$$\begin{aligned} r_{FR_{t+n}}^{(n)} &= \sum_{j=1}^{n} r_{FR,t+j} \\ &= \alpha_t'(n)\left(x_{A,t+n}^{(n)} + \frac{1}{2}\sigma_A^2\right) - \frac{1}{2}\alpha_t'(n)\,\Sigma_{AA}\,\alpha_t(n) - x_{L,t+n}^{(n)} \end{aligned} \qquad [2.8]$$

In this setup for a long-term portfolio problem the influencing factors are variances and covariances. In a random fashion there is no reason to adjust the portfolio weights over time.
In the following two different portfolios are derived. In the first case it is assumed that there is no risk-free asset and that the investor is interested in the portfolio that minimizes variance. This leads to the LHPF respectively to the GMV portfolio, latter considering the asset-only investor.[5] In the second case it is expected that there exists a truly risk-free asset. Furthermore I work in this scenario with excess returns over liabilities on asset level. Following this assumption it is possible to calculate the tangency portfolio - which is the portfolio maximizing the Sharpe ratio and that has no loading on the risk-free asset.

[5] This will let us gain information about differences between an asset-liability and an asset-only investor and about horizon effects.

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2.5 Case 1: LHPF respectively GMV portfolio

The GMV portfolio denotes the portfolio with the lowest variance for an asset-only investor. In the following the LHPF is derived, which is the portfolio with the lowest variance for an asset-liability investor. Calculation for the asset-only investor differs only slightly, but leads to almost the same result.
The assumed investor chooses the portfolio weights α_t(n) that minimize variance per period subject to a required expected return per period.

This portfolio problem can be described as

$$\min_{\alpha} \frac{\frac{1}{2}Var_t\left(r_{FR,t+n}^{(n)}\right)}{n} \quad [2.9]$$

subject to $\frac{E_t\left(r_{FR,t+n}^{(n)}\right)+\frac{1}{2}Var_t\left(r_{FR,t+n}^{(n)}\right)}{n} = \mu_{FR}$
where μ_{FR} is the required return per period.

Straightforward the Lagrange can be written as

$$L = \frac{\frac{1}{2}Var_t\left(r_{FR,t+n}^{(n)}\right)}{n} + \lambda \left(\mu_{FR} - \frac{E_t\left(r_{FR,t+n}^{(n)}\right)+\frac{1}{2}Var_t\left(r_{FR,t+n}^{(n)}\right)}{n}\right)$$

whereby the Lagrange multiplier is denoted as λ.

The solution for the mean-variance problem can be expressed as

$$\alpha_t(n) = \lambda\, (\Sigma_{AA}(n))^{-1} \left(\mu_t^{(n)} + \frac{1}{2}\sigma_A^2\right) - (\Sigma_{AA}(n))^{-1} (\sigma_{AL}(n)) \quad [2.10]$$

The LHPF is obtained by letting λ approach zero.

$$\alpha_{LHPF}(n) = - \Sigma_{AA}(n)\, \sigma_{AL}(n) \quad [2.11]$$

The calculation for an asset-only investor (GMV portfolio) leads to the same equation [2.11] except for that the variance-covariance matrix between assets and liabilities $\sigma_{AL}^{(n)}$ must be replaced with the variance-covariance matrix between risky assets and the benchmark asset $\sigma_{Arnom}^{(n)}$.

2.6 Case 2: Tangency portfolio for asset over liability returns

It is now assumed that the investor wants to maximize the portfolio's Sharpe ratio, instead of minimizing variance. The maximization of the Sharpe ratio describes the risk-return relation that leads to the portfolio with the highest expected return per unit of risk and therefore is the most risk-return efficient portfolio.
Moreover, it is now assumed that the investor has a risk-free asset available, denoted as $r_{rnom}^{(n)}$. Furthermore, I work now with asset over liability returns assuming an asset-liability investor.[6] I denote the excess asset return as r_{EA} and the 3m CHF Libor rate is expected to be the risk-free, in this set-up. This risk-free asset can be seen as a zero-coupon bond. Therefore, the investor has now n+1 assets available for investment. However, in the tangency portfolio solution the investor is not invested in the risk-free asset.

[6] The difference to before is that liability returns are subtracted from returns for each asset separately instead of on a portfolio level.

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Following Campbell and Viceira (2005) the portfolio problem can now be stated by

$$\min_{\alpha} \frac{1}{2} \frac{Var_t\left(r_{EA,t+n}^{(n)} - n\, r_{rnom}^{(n)}\right)}{n} \qquad [2.12]$$

Subject to $\frac{E_t\left(r_{EA,t+n}^{(n)} - n\, r_{rnom}^{(n)}\right) + \frac{1}{2} Var_t\left(r_{FR,t+k}^{(n)} - r_{rnom}^{(n)}\right)}{n}$

The solution to the mean-variance problem can be expressed as

$$\alpha_t\,(\mathrm{n}) = \lambda_{rf}\, \Sigma_{EA}^{-1}(\mathrm{n}) \left(E_t\left(r_{EA,t+1}^{(n)} - n\, r_{rnom}^{(n)}\iota\right) + \frac{1}{2}\sigma_{EA}^2(n)\right) \quad [2.13]$$

Now it is possible to derive the tangency portfolio weights. Therefore the portfolio weights α_t (n)' ι is set equal one. This can be expressed as

$$\lambda = \frac{1}{\left(E_t\left(r_{t+1}^{(n)} - n\, r_{rnom}^{(n)}\iota\right) + \frac{1}{2}\sigma_r^2(n)\right)'\left(\Sigma_{EA}^{-1}\ (n)\right)'\iota} \qquad [2.14]$$

So that the portfolio weights of the tangency portfolio can be stated as

$$\alpha_t\,(\mathrm{n}) = \frac{1}{\left(E_t\left(r_{t+1}^{(n)} - n\, r_{rnom}^{(n)}\iota\right) + \frac{1}{2}\sigma_r^2(n)\right)'\left(\Sigma_{EA}^{-1}\ (n)\right)'\iota}\, \Sigma_{rr}^{-1}(\mathrm{n}) \left(E_t\left(r_{t+1}^{(n)} - n\, r_{rnom}^{(n)}\iota\right) + \frac{1}{2}\sigma_r^2(n)\right) \qquad [2.15]$$

3. Analyzing inter-temporal relations

In this chapter data for the analysis is specified and the estimation results are presented. Then, based on the return dynamics implied by the VAR(1) model, the long-run variance and correlation characteristics of assets and liabilities are analyzed. Hereby, I firstly evaluate conditional volatilities over the investment horizon within asset classes respectively of asset returns over liability returns. Then conditional correlations between returns on each of the asset classes and returns on liabilities are analyzed across the investment horizon.

3.1 Data

This analysis includes the typical asset classes in which a Swiss life insurance company invests.[7] The sample period covers 19 years, from January 1, 1994 to December 31, 2012. For stocks it is distinguished between stocks Switzerland and stocks global. Stocks Switzerland are represented by the Swiss Performance Index; this total return index includes almost all listed companies in Switzerland and is calculated since June 1987. It is a commonly used index representing the broader Swiss stock market. Monthly log returns are considered for the analysis. Returns for stocks global are based on the MSCI global equity index, which provides coverage for over 70 countries. This total return index is in USD and as mentioned above: index values are converted into CHF before the calculation of monthly log returns. Monthly log returns for Swiss government bonds are based on the Bloomberg index for Swiss government bonds with maturities of ten years and more. Monthly log returns for bonds global are obtained from the JPM Aggregate Bond Index.
Private equity returns are based on the LPX50 total return equity index, which is a global index covering the 50 largest private equity companies.[8] It is further diversified across investment- and financing styles and vintage years. Monthly log returns are considered. Monthly log hedge fund re-

[7] Only direct real estate investments are not considered. Because of their local nature it is difficult to get return series that are comparable with other asset classes for such a specific mean-variance analysis.
[8] That a company gets included into this index it has to fulfill specific liquidity constraints.

turns are obtained from the HFRI fund of funds composite. This index represents the whole industry and does not account for different hedge fund styles. Various studies show that risk and return characteristics vary between different fund strategies. Since hedge funds are included as one asset class I do not further discuss differences between fund styles. Commodity returns are based on the S&P GSCI Commodity total return index. This tradable index compromises commodities from all commodity sectors and is world-production weighted. It is listed at the Chicago Mercantile Exchange in USD and monthly log returns are used for the analysis. Log returns of listed real estate investments are based on the EPRA/NAREIT Developed Index. This total return index includes developed markets – instead of developed and emerging markets. It represents publicly property companies that trade on several global exchanges.

Liability returns are replicated with the yield of the 20-year Swiss Confederation bond. Expecting evenly distributed business volume over time, the average yield rate over the last 10-years should model the interest expenses fairly well.[9] The three state variables included in the analysis are based on US data, due to their liquidity and availability. These three state variables are commonly used in literature to predict asset returns. Credit spread is calculated as the difference between the log Baa US zero-coupon and the log of the ten-year US T-bill. Yield spread is calculated as the difference between the ten-year zero-yield T-Bill and the 3m T-bill. The calculation of spreads is based on monthly yields. Finally, the log of the dividend yield of the S&P Composite is used. Data for Swiss government bonds and for bonds global are from Bloomberg; all other data series except for private equity are from Datastream.[10]

3.2 VAR estimation results

Table 1 provides annualized average means of excess gross returns over the benchmark asset for each asset class and for liabilities. Mean log excess returns are adjusted by adding one-half their variance so that they reflect mean gross returns, except for the dividend-price ratio. Stocks Switzerland and bonds Switzerland performed over the considered period with an average excess rate over the benchmark asset of 6.6% respectively 4.3%. The average return rates of stocks global and bonds global are due to the development of the CHF/USD exchange rate considerably lower compared to the Swiss indices. Private equity and listed real estate performed both with an average gross excess return rate of over 6%, but both and especially private equity suffered a high volatility. Hedge funds performed with an average excess rate of 1.95% and also had a comparably high volatility. Commodities performed in average almost 1% better than hedge funds but also more volatile, which leads to the lowest Sharpe ratio of the considered asset classes. However, they performed well until the financial crisis, but then suffered high losses in 2008 and 2009.[11] Liabilities have an average gross excess return of 2.9%. Volatility is extremely low, because for an insurer liability expenses are driven by the past development of the interest rate (they must be understood as a weighted interest yield over the last years and therefore adjust only slowly and with a timely delay to interest yield developments). Noteworthy is that over the considered period not all asset classes performed well enough to cover liability expenses. Therefore, bonds global, hedge funds and commodities have to exhibit high diversification benefits for getting included into the portfolio allocation.

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9 Yields of the ten-year bond of the Swiss Confederation are available since 1988. Effectively the average yield covers ten years as per early 1998. Since the yield level decreased over the last years of the sample period the average level represents the effective interest expenses. Without loss of generality interest rates for the first few years of the sample period can effectively be calculated as an average of a shorter period. Furthermore, law sets a minimum interest rate for running collective life insurance contracts; the constructed yield series never falls below this threshold. The by law specified minimum BVG interest rate and its history can be found at www.bsv.admin.ch

10 Data for private equity was provided by LPX Group (data is not publicly available and was received upon request). At this point I want thank LPX Group for providing this data to me.

11 The exchange rate development also affects the return rates of private equity, hedge funds, commodities and listed real estate as the considered indices are listed all in USD.

Table 1: Descriptive Statistics

This table reports - based on monthly log excess returns over the risk free benchmark asset - annualized means and standard deviations (STD), Sharpe ratios and monthly minimum and maximum values. Mean log excess returns are adjusted by adding one-half their variance so that they reflect mean gross returns, except for dpr.

Assets:	mean	STD	SR	min	max
rnom	1.59	0.36			
stch	6.60	17.23	0.38	-20.28	17.43
stgl	4.08	18.22	0.22	-18.61	12.83
boch	4.31	6.45	0.67	-4.44	10.23
bogl	2.36	8.09	0.29	-6.73	11.00
preq	6.61	26.86	0.25	-35.19	27.65
hefu	1.95	12.52	0.16	-12.07	10.25
comm	2.94	23.23	0.13	-31.87	17.19
reli	6.28	19.64	0.32	-29.54	18.04
State Variables:					
csp	2.40	0.25		0.11	0.51
ysp	1.66	0.35		-0.06	0.32
dpr	1.87	0.14		0.09	0.29
Liabilities:					
liab	2.67	0.29		0.04	0.35

Table 2 provides the VAR(1) estimations for the specified model with t-statistics in parentheses and Table 3 reports the covariance structure of the innovations: percentage standard deviations and correlations based on the monthly excess returns over the benchmark asset. The process generating parameters and their evolution over time – respectively means, variances and covariances – are set correspondingly to their historically estimated values. The coefficients are estimated by least squares applied separately to each of the equations.

Excess returns on stocks Switzerland are not significantly explained by any of the state variables. Usually, yield spreads widen in recessions, which leads to lower stock returns in the subsequent period. Excess returns on stocks global are explained by the yield spread. Excess returns on stocks Switzerland can help to forecast excess bond returns Switzerland. Noteworthy is that bonds global are not significantly predicted – at least on a 0.05 level – by any of the variables. A possible reason for that could be the development of the exchange rates (since index values have been converted before return calculation), therefore bond returns are affected by the exchange rate development and state variables can only weakly explain the by the exchange rate development affected returns, at least in this case. Private equity and hedge funds are significantly related to the yield spread. Listed real estate and commodities are not significantly explained by any of the state variables, but both have a high exposure to private equity. For liabilities the short-term interest rate and the credit spread are significant predictors.

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Table 2: VAR Estimation Results

This table reports the estimation results of the VAR(1) specified in [2.1] for monthly excess returns for assets and liabilities over the benchmark asset. In the last column R squared values are provided. T-statistics are reported in cursive.

	1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)	12)	13)	R squared
1) rnom	0.9491	-0.0004	0.0001	0.0001	-0.0006	0.0005	-0.0002	0.0002	0.0002	-0.0453	-0.0174	-0.0528	-0.0327	0.97
	62.8560	*-0.7398*	*0.2060*	*0.0738*	*-0.6827*	*1.6821*	*-0.2932*	*0.6920*	*0.5776*	*-1.2728*	*-1.5410*	*-2.3329*	*-1.5250*	
2) stch	-4.9616	-0.0288	0.0840	0.0929	0.0166	0.0757	-0.0979	-0.0487	0.0285	15.9170	-9.4821	-6.2644	10.9470	0.12
	-1.1051	*-0.1616*	*0.3474*	*0.4509*	*0.0506*	*0.7462*	*-0.2966*	*-0.8602*	*0.2817*	*1.1598*	*-1.9398*	*-0.7686*	*1.4316*	
3) stgl	-3.0791	0.1148	-0.1904	0.2099	-0.0131	0.2049	-0.0241	-0.0800	-0.0033	17.5520	-12.6090	0.2352	16.5560	0.17
	-0.6246	*0.9700*	*-0.9245*	*1.0141*	*-0.0424*	*2.1479*	*-0.0937*	*-1.4134*	*-0.0349*	*1.2616*	*-2.8136*	*0.0285*	*2.3240*	
4) boch	-1.1803	0.0920	-0.0149	-0.1394	0.2093	-0.0541	-0.0999	-0.0089	-0.0475	0.8113	1.7838	-1.4583	1.0974	0.11
	-0.5865	*1.9983*	*-0.2463*	*-1.3422*	*1.8571*	*-1.9092*	*-1.1966*	*-0.3728*	*-1.1728*	*0.2051*	*1.1714*	*-0.5510*	*0.4500*	
5) bogl	0.0704	0.0374	-0.0097	0.0125	-0.0215	0.0234	-0.0365	-0.0393	-0.0434	2.0062	-3.4476	0.9653	4.6995	0.06
	0.0244	*0.5938*	*-0.1182*	*0.1082*	*-0.1383*	*0.5222*	*-0.4236*	*-1.4023*	*-0.8017*	*0.4709*	*-1.8341*	*0.3137*	*1.6366*	
6) preq	-2.1320	0.0836	-0.2032	0.4602	-0.4291	0.3931	0.0743	-0.0270	0.0767	21.7730	-13.5210	-0.8331	24.9220	0.25
	-0.3046	*0.5091*	*-0.7383*	*1.2894*	*-0.9164*	*2.9382*	*0.1952*	*-0.3202*	*0.4934*	*1.0378*	*-2.1301*	*-0.0634*	*2.5551*	
7) hefu	1.0684	0.0600	-0.0354	-0.0269	-0.1318	0.1311	0.0546	-0.0743	0.0055	1.2271	-7.4159	3.1670	9.3335	0.16
	0.2922	*0.6762*	*-0.2556*	*-0.1904*	*-0.6739*	*1.9661*	*0.4131*	*-1.8290*	*0.0783*	*0.1766*	*-2.6601*	*0.6811*	*2.1276*	
8) comm	-0.0068	-0.1286	-0.0827	-0.2046	-0.1332	0.2237	0.0979	-0.0156	0.1181	-9.2973	-2.6738	-1.6975	8.1096	0.11
	-0.0010	*-0.7899*	*-0.3758*	*-0.8293*	*-0.3470*	*2.1532*	*0.3676*	*-0.2190*	*0.8821*	*-0.7177*	*-0.5124*	*-0.2155*	*0.9050*	
9) reli	-4.9804	0.0597	-0.0223	0.3043	0.3009	0.2214	-0.3569	0.0182	0.0128	-0.5318	-7.9657	1.4273	9.9599	0.18
	-1.0069	*0.5480*	*-0.1194*	*1.5468*	*0.9745*	*2.6479*	*-1.5451*	*0.3140*	*0.1366*	*-0.0416*	*-1.8310*	*0.1763*	*1.3264*	
10) dpr	-0.0069	-0.0003	0.0006	0.0001	-0.0002	-0.0002	0.0000	-0.0001	0.0000	0.9423	0.0172	-0.0098	-0.0374	0.97
	-1.2060	*-1.5509*	*2.2041*	*0.4388*	*-0.6450*	*-1.9870*	*0.0041*	*-1.8437*	*-0.1678*	*43.9200*	*2.4694*	*-0.7832*	*-3.1803*	
11) ysp	0.0385	-0.0014	0.0011	0.0004	-0.0017	0.0003	0.0000	0.0001	0.0006	-0.0320	0.9426	0.1251	0.0023	0.94
	1.4222	*-2.2466*	*1.2345*	*0.3103*	*-1.2444*	*0.5528*	*-0.0244*	*0.5292*	*1.0293*	*-0.6222*	*49.7660*	*3.7871*	*0.0713*	
12) csp	-0.0258	-0.0001	-0.0012	-0.0005	0.0009	-0.0003	0.0004	0.0000	-0.0001	-0.0442	0.0089	0.9309	-0.0612	0.95
	-1.6751	*-0.3252*	*-2.0671*	*-0.6168*	*0.9143*	*-1.2020*	*0.7427*	*0.2515*	*-0.3611*	*-1.2685*	*0.7642*	*39.3880*	*-2.7322*	
13) liab	0.0559	0.0004	-0.0001	-0.0001	0.0006	-0.0005	0.0002	-0.0002	-0.0003	0.0520	0.0173	0.0522	1.0373	0.96
	3.7063	*0.7286*	*-0.1501*	*-0.1014*	*0.7051*	*-1.6480*	*0.2684*	*-0.6975*	*-0.6483*	*1.4617*	*1.5377*	*2.3084*	*48.5490*	

The last column of Table 2 provides the R squared values. It can be noted that excess returns for bonds (11% resp. 6%) and commodities (11%) are difficult to explain. Moreover are stocks Switzerland (12%) only weakly explained. However, it should be considered that on one hand the given R squared values are based on monthly return data, therefore the corresponding R squared value are on an annual basis considerably higher. On the other hand, even low R squared values can be economically meaningful for mean-variance investors, if state variables are highly persistent. The high slope values of the predictive state variables on their own lags indicate that they are highly persistent: dividend-price ratio 0.94, credit spread 0.94 and yield spread 0.93. Furthermore, the system is stable; the maximal eigenvalue is 0.991, but close to being integrated of order one.

Table 3 provides the covariance structure of the innovations in the VAR system with percent volatilities on the diagonal and correlations coefficients off-diagonal. Unexpected log excess stock returns are moderately to strongly positive correlated with unexpected log excess returns on private equity and listed real estate; moderately positive with bonds global, hedge funds and commodities. Correlation between unexpected excess returns on stocks and the dividend-price ratio is moderately negative and weakly moderately negative with credit spread. Coefficients are higher for stocks global than for stocks Switzerland with each of the mentioned asset classes respectively state variables. All other correlations – with bonds Switzerland, yield spread and liabilities are fairly low. Furthermore, un-

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expected excess returns of stocks Switzerland are highly positive correlated with unexpected excess returns of stocks global. Unexpected log excess bond returns are positively correlated[12] with unexpected log returns on liabilities, with shocks to the credit spread and as well are unexpected log excess returns of bonds Switzerland correlated with them of bonds global. Furthermore, they are negatively correlated with shocks to the yield spread, whereby the correlation coefficient is higher for bonds Switzerland than for bonds global. Correlation with hedge funds is only slightly positive with bonds Switzerland and strongly positive with bonds global. Correlation with commodities is negative for bonds Switzerland and positive for bonds global, but on a fairly low level. Moreover, unexpected log excess returns on private equity are positive correlated with unexpected log returns on listed real estate[13] and negative to shocks in the dividend-price ratio and the credit spread. Unexpected log excess hedge fund returns are on a low level positive correlated with unexpected log excess returns on commodities, on a moderate positive level with listed real estate and negative correlated with shocks in all three considered state variables and with unexpected excess log returns on liabilities, as well on a fairly low level.

Table 3: VAR Error Correlation Matrix

Monthly percent standard deviations of residuals are given on the diagonal, off-diagonal figures refer to cross-correlations of residuals. The provided error correlation matrix relates to the above provided VAR(1).

	1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)	12)	13)
1) rnom	0.0164												
2) stch	0.0647	4.6667											
3) stgl	0.1081	0.7837	4.8199										
4) boch	-0.3168	-0.0311	-0.1136	1.7622									
5) bogl	-0.0194	0.1349	0.3728	0.2963	2.2726								
6) preq	0.1174	0.6587	0.8103	-0.1222	0.1305	6.7630							
7) hefu	0.0593	0.2748	0.5621	0.0163	0.7949	0.3558	3.3179						
8) comm	0.1120	0.2176	0.4037	-0.0680	0.1585	0.3834	0.2945	6.3491					
9) reli	0.0277	0.6015	0.7829	0.0286	0.3708	0.6912	0.4771	0.3542	5.0741				
10) dpr	0.0680	-0.5220	-0.5297	-0.0454	-0.0657	-0.4949	-0.0932	-0.1704	-0.4565	0.0064			
11) ysp	-0.0089	0.0548	0.0711	-0.3892	-0.2459	0.1323	-0.0094	-0.0452	-0.0334	0.0577	0.0243		
12) csp	-0.0803	-0.2783	-0.3823	0.2298	0.1790	-0.4780	-0.1064	-0.2779	-0.4002	0.2463	-0.1668	0.0160	
13) liab	-0.9997	-0.0643	-0.1065	0.3126	0.0176	-0.1155	-0.0605	-0.1127	-0.0297	-0.0694	0.0141	0.0806	0.0163

Table 2 and 3 provide information about the return dynamics; future period returns depend on past shocks, which are again contemporaneously correlated with returns. For example returns on stocks are positively forecasted by the dividend-price ratio, but the contemporaneous correlation between stocks and the dividend-price ratio is negative. Straightforward, dividend-price ratio induces negative first-order auto-covariance in the subsequent one-period return. This phenomenon is known as mean-reversion. Of course, to analyze mean-reversion respectively mean- aversion characteristics over the horizon all return forecasting variables within the model have to be considered. That is the focus of the subsequent subchapter.

[12] For bonds global correlation with liabilities is almost zero.
[13] Both have a high exposure to stocks.

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Implications over the investment horizon

To further investigate the importance of asset-liability management for long-term investment decisions I now analyze volatilities and correlations between assets and liabilities across the horizon. The long-term return dynamics follow from the VAR(1) parameters.
Figure 1 gives the annualized conditional volatilities for asset returns over liability returns for different investment horizons, up to 20 years (240 months). Moreover, the annualized conditional volatilities of the benchmark asset and of liabilities are provided. Volatility of the 3m CHF Libor rate increases with an increasing investment period, up to almost 3% for a 20-year horizon. In general, investments in the short-term rate get riskier in the long-term due to reinvestment risk. Liabilities have a very low and also constant volatility over the investment horizon. This is caused due to their nature since they reflect a weighted average of the interest yield development over the past years and therefore generally enjoy low volatility.

Risk for stocks Switzerland and for stocks global develops very similar over the investment horizon, after a hump-shaped pattern conditional volatility slightly decreases. This slightly mean-reversion behavior is caused due to that a negative shock in the dividend-price ratio induces a positive shock to stock returns (negative correlation) and therefore positive innovations in the dividend-price ratio, which have a negative effect on the contemporaneous excess stock return. However mean-reversion is only weak for stocks because on the one hand the effect is dampened due to positive correlation with shocks in the yield spread, and on the other hand volatility of liabilities slightly increases over the horizon, which also dampens the mean-reversion of the excess stock returns over liability returns.
As well for bond over liability returns volatility is comparatively constant over the horizon, while it is lower for bonds global than for bonds Switzerland. This because bonds Switzerland are replicated by a government bond index and therefore enjoy higher credit ratings compared to bonds global represented by an aggregated bond index. Risk remains more or less on the same level over the investment horizon for hedge funds and commodities, slightly increasing over the first few years for hedge funds and slightly decreasing for commodities. For private equity and listed real estate annualized conditional volatility slightly decreases with an increasing investment horizon.

Mean-reversing characteristics for stocks and bonds are weaker compared with previous studies – for example [9] or [11]. This can be explained by considering several facts. Firstly, I work with a different data set. In the meantime the macro-economic situation seems to have changed caused by the financial crisis, lower interest rates and more volatile markets. Therefore the state variables are not able to explain as much of the dynamics as in previous studies. Furthermore, the exchange rate development which influences the CHF values of USD listed indices may also be a reason for the weaker capturing of this dynamics. To sum up, the sample period is shorter, there are more and more specific assets included and overall predictive power seems to be lower due to the latest development of the macro-economic environment. And secondly, results are only weakly comparable with previous studies since I work with nominal rates instead of real rates. However, this approach best describes the investment problem for a Swiss life insurance company, which cares about total nominal returns.

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Figure 1: Volatilities over the investment horizon

This figure provides annualized conditional standard deviations for asset returns over liability returns. On the x-axis the holding period is given in months (up to 240). The y-axis provides annualized volatilities. The graph left on the top gives annualized conditional standard deviations of the benchmark asset and of liabilities.

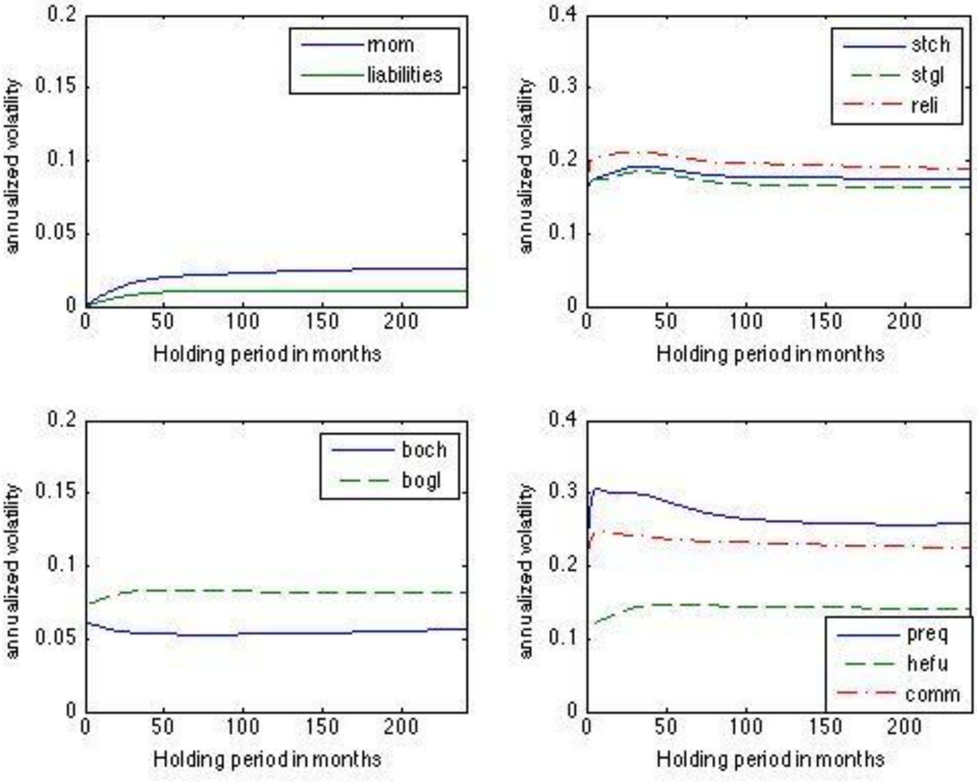
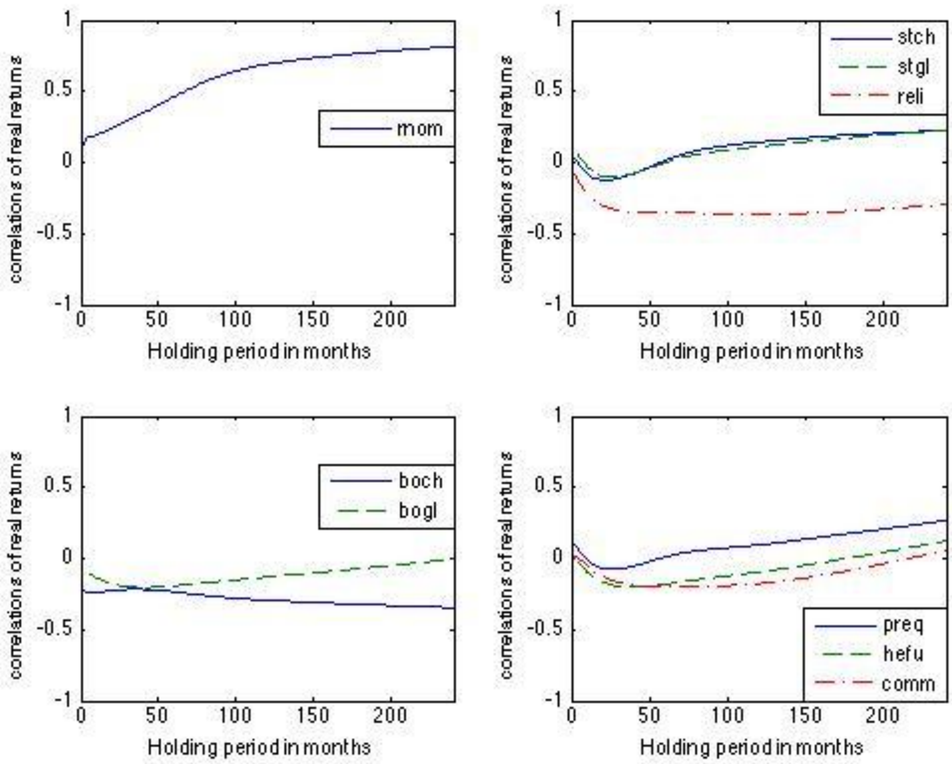


Figure 2 provides correlation coefficients between returns of each of the asset classes and returns of liabilities. Correlation between the short-term rate and liabilities increases strongly for longer investment periods, up to ten years to a level of almost 0.8. For a longer investment horizon it further increases up to almost 0.86 for a 20-year horizon. Correlation between liabilities and bonds Switzerland is lower than with bonds global and the gap widens with an increasing investment horizon. It decreases with an increasing investment horizon for bonds Switzerland and also decreases over the first few years for bonds global before correlation recovers. Overall is correlation between liabilities and bonds global always around zero and therefore is the lowest compared with all other considered asset classes. Between stocks and liabilities correlation shows a hump-shaped pattern in the short-run while it gets down from around zero to -0.2, but with a longer investment horizon correlation increases and even gets positive up to almost 0.25 for a 20-year horizon. Correlation coefficients with liabilities are almost equal for stocks Switzerland and stocks global. Between liabilities and listed real estate correlation decreases strongly at the very beginning and then remains almost constant equal at around -0.3 with an increasing investment horizon. Correlation between private equity and liabilities is comparably with correlation between stocks and liabilities, which can be explained with their high exposure to stocks. It is hump-shaped in the short-run getting slightly negative for a short investment period before it increases finally to around 0.25 for a 20-year horizon. For hedge funds and commodities correlation with liabilities decreases and remains slightly negative but gets for the 20-year investment period marginally positive.

Figure 2: Correlation between asset returns and liability returns

This figure provides correlation coefficients between returns of each of the asset classes and liabilities for an increasing investment horizon up to 20 years (240 months). On the x-axis the holding period is provided in months. The y-axis gives correlation coefficients.



The analysis of inter-temporal relations between assets and liabilities leads to important results for portfolio allocation. The analysis of volatilities of each of the considered asset classes over liabilities shows that risk differs between each of the asset classes and especially important that within an asset class risk may be different for a short-term investor compared with a long-term investor. Variations in risk across the horizon are weaker than in previous studies but I found mean-reversing characteristics for stocks, listed real estate and private equity.[14] Moreover, correlations between asset classes and liabilities and accordingly between asset classes differ for different investment horizons. These return dynamics drive asset allocation implications. This is what the subsequent chapter is focusing on.

4. Portfolio implications

This chapter compares the minimum-variance portfolio for the asset-liability and the asset-only investor. Moreover portfolio weights are provided for the tangency portfolio for the asset-liability investor and horizon effects are emphasized. Finally implications on strategic asset allocation are critically discussed.

[14] Classical mean-variance theory would further work with results provided in Table 1. However, by taking out predictability the resulting dynamics (see Figure 1 and 2) can be expected to have implications on the portfolio task.

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4.1 LHPF and GMV

Table 4 gives portfolio weights for the LHPF and for the GMV portfolio for investment horizons of one, five, ten and 20 years. The LHPF is the leftmost point on the efficient frontier for an asset-liability investor and therefore the portfolio with the lowest variance.[15]

Figure 4: LHPF and GMV Portfolio

This table provides portfolio weights for the LHPF and the GMV-portfolio for a 1, 5, 10 and 20 year investment period.

LHPF					GMV				
period (y)	1	5	10	20	period (y)	1	5	10	20
rnom	0.94	0.82	0.81	0.80	rnom	0.98	0.90	0.88	0.87
stch	-0.01	-0.04	-0.04	-0.05	stch	0.00	-0.03	-0.04	-0.04
stgl	0.02	0.01	-0.01	-0.01	stgl	0.01	0.03	0.02	0.02
boch	0.06	0.16	0.19	0.22	boch	0.03	0.12	0.14	0.15
bogl	-0.01	-0.02	-0.04	-0.06	bogl	-0.01	-0.01	-0.02	-0.02
preq	0.00	0.01	0.01	0.00	preq	0.00	0.00	0.00	-0.01
hefu	0.01	0.04	0.06	0.06	hefu	0.00	0.00	0.01	0.01
comm	0.00	0.00	0.00	0.00	comm	0.00	-0.01	-0.01	-0.01
reli	0.00	0.01	0.03	0.05	reli	0.00	0.01	0.02	0.03

While comparing the LHPF with the GMV portfolio two effects can be recognized: the influence of the liabilities and the impact of the investment horizon. For a one-year investment horizon the asset-liability investor allocates a smaller proportion of wealth to the 3m CHF Libor rate, slightly less to stocks Switzerland, slightly more to stocks global and to hedge funds and twice as much to bonds global compared with the asset-only investor. There are various effects that lead to this allocation and therefore are responsible for the differences between the asset-liability and the asset-only investor. However, the two main differences in portfolio weights are the lower share allocated to the benchmark asset and the higher share allocated to bonds Switzerland from the point of view of the asset-liability investor. This because excess return of bonds Switzerland over liabilities have a comparably low volatility and therefore get a proportional higher share in the minimum-variance portfolio. Therefore the asset-liability investor allocates more to bonds at the expense of the benchmark asset. At this point it must be considered that the benchmark asset in this application gets that high weight due to its low volatility. With an increasing investment horizon both the asset-liability and the asset-only investor lower their weight in the 3m CHF Libor rate, but the decrease is stronger for the asset-liability investor compared with the asset-only investor.[16]

The amount allocated to stocks Switzerland decreases for both and no differences can be recognized. For stocks global the weight is also very low, gets slightly negative for the asset-liability investor and remains slightly positive for the asset-only investor with an increasing investment horizon. The weight allocated to bonds Switzerland increases for both – but in absolute volume stronger for the asset-liability investor (up to 22% compared with up to 15% of total portfolio weight). The weight of bonds global develops differently when the investment horizon increases – it decreases for the as-

[15] Correspondingly the portfolio with the lowest variance for the asset-only investor is the GMV portfolio.
[16] The decrease for the asset-liability investor from a 1y investment horizon to a 20y investment horizon is -15% (respectively -11% for the asset-only investor).

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set-liability investor and remains more or less stable for the asset-only investor. This because excess returns over liabilities are for bonds global very low and therefore get stronger negative. Private equity and commodities are for both minimum-variance portfolios not an important asset class as they have a high volatility. And finally hedge funds and listed real estate investments get more important with an increasing investment horizon for the asset-liability investor. For example with a 20-year investment horizon the asset-liability investor allocates 6% of his assets to hedge funds compared with 1% for the asset-only investor. And the share invested in listed real estate for the 20-year investment horizon is 5% for the asset-liability investor compared with 3% for the asset-only investor. From a return perspective hedge funds are unfavorable. The higher weight can be explained by considering volatilities and correlations. Volatility remains comparable low, but is still much higher than for bonds. But hedge funds have a comparably low, and even over the horizon slightly decreasing correlation with bonds Switzerland, which makes them a good diversifier. It can be seen that portfolio implications are different if considering liabilities and furthermore that the investment horizon affects the minimum-variance portfolio weights. Latter because risk varies across the horizon.

4.2 Tangency portfolio

I now consider asset returns over liability returns on asset class level instead of on portfolio level. The tangency portfolio is the portfolio with the most favorable risk-return relation. The difference between the asset-liability and the asset-only investor is that the latter has to cover liability expenses and therefore requires a higher return on his assets.[17] It is important to see the difference to before. Therefore in this setup different – but at least as important – implications can be expected. Table 5 provides portfolio weights for the tangency portfolio for an asset-liability investor. The Sharpe ratio is very low compared with the Sharpe ratios provided in Table 1. This because excess returns are considerably lower due to that asset returns are reduced by liability returns respectively liability expenses. Noteworthy is that the Sharpe ratio increases with an increasing investment horizon, which means that the risk-return tradeoff gets slightly more favorable for an investor with a longer horizon, which can be attributed to the proven mean-reversing effects and of course is also affected by the specific portfolio composition.

The weights allocated to stocks Switzerland are the same for an investment period of five, ten and 20 years; at around 38% of total wealth. Anyway the share allocated to stocks Switzerland is almost 10% higher for an investor with a one-year investment horizon. That can be explained by considering volatilities and correlations. Volatility is about the same for a short investment horizon compared with a horizon of five years. It then gets lower with a further increasing investment horizon (mean-reversion). Therefore, most of this large difference can be attributed to correlation coefficients. For a one-year horizon correlation between stocks Switzerland and bonds global is lower than for longer horizons. Furthermore, but less important, correlation between stocks Switzerland and hedge funds and also between stocks Switzerland and private equity increases as well from a one-year to a five-year horizon and then remains on a constant – only marginally decreasing - level for a further increasing investment horizon. This makes stocks in the short-term perspective a better risk diversifier. Coefficients with all other asset classes are more or less constant across the time axis. Especially the diversification attribute between stocks and bonds may be responsible for the higher proportional allocation to stocks Switzerland. Anyway, correlation between stocks Switzerland and liabilities increases with an increasing horizon which makes them a better hedge against liabilities. Overall the first effect is stronger in this case.

[17] He requires a higher return because he has to cover liability expenses.

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Table 5: Tangency Portfolio

This table provides portfolio weights for the tangency portfolio for a 1, 5, 10 and 20 year investment period. Calculation is based on VAR coefficients capturing dynamics of assets and liabilities. Values are given in percent.

Investment period (y)	1	5	10	20
liab	4.25	4.25	4.25	4.25
Sharpe Ratio	0.13170	0.13420	0.13510	0.13540
stch	46.99	38.89	38.19	38.22
stgl	-60.92	-45.31	-42.03	-42.16
boch	110.17	100.76	96.72	92.78
bogl	-28.79	-14.41	-12.23	-10.58
preq	9.64	9.02	9.61	10.76
hefu	2.65	-2.94	-2.76	-1.05
comm	0.95	0.60	0.91	1.94
reli	19.32	13.39	11.59	10.08

The share allocated to bonds Switzerland gets constantly lower with an increasing horizon, it decreases from 110% of the portfolio for a one-year horizon to 94% for a 20-year horizon. That means that they become less important for a long-term investor. Of course bonds Switzerland remain the most important asset class for all considered investment horizons because of their favorable risk-return ratio. With an increasing horizon correlation decreases between bonds Switzerland and stocks Switzerland. Furthermore, correlation between bonds Switzerland and liabilities decreases constantly with an increasing horizon, which makes them a comparably bad liability hedge and therefore its portfolio weight decreases with an increasing investment horizon. The shares allocated to stocks global and bonds global are both negative, but with an increasing horizon the negative proportional shares decreases, which means that they are more interesting for a long-term asset-liability investor. This can be explained by the slightly mean-reversing effects and moreover by considering correlations with other asset classes. Correlation with traditional asset classes is except for the very short-run almost constant. However, correlations with all alternative asset classes decrease, which makes stocks global and bonds global from a risk diversification perspective more important for longer investment periods.

Hedge funds and commodities are no important asset classes in the tangency portfolio view, for none of the considered horizons. Hedge funds are positively included only for the one-year investment horizon and get a negative weight for all other considered longer investment periods. They have a high exposure to stocks global and to bonds global and as their shares increase with an increasing investment horizon the share of hedge funds decreases. Commodities are below 1% for a one, a five and a ten-year horizon and are included with almost 2% for the 20-year investment horizon. This because they become a better risk diversifier in the long-run since they have a lower correlation with stocks Switzerland and bonds Switzerland. Private equity is included in the portfolio with around 10% at all considered horizons. This can be attributed on one hand to their favorable risk-return relation and on the other hand to their low correlation with bonds Switzerland, which makes them a good diversifier. Finally, the share allocated to listed real estate decreases with an increasing investment horizon, from around 20% down to around 10%. This can be attributed to their decreasing

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correlation with liabilities, which makes them a worse diversifier.

4.3 Portfolio implications

While comparing the minimum-variance with the tangency portfolio for the asset-liability investor various differences can be recognized. With an increasing horizon the relative share of bonds Switzerland increases in the LHPF while it decreases in the tangency portfolio. For bonds global the opposite applies. The weight of stocks Switzerland decreases in both with an increasing horizon. For stocks global the share decreases in the LHPF and increases in the tangency portfolio, anyway in latter the share is negative for all considered investment period. The relative share of private equity and commodities is more or less stable in both contexts. And finally the weight of hedge funds and real estate increases in the LHPF, while it decreases in the tangency portfolio with an increasing horizon. These effects can be attributed to the interplay of various inter-temporal dynamics. However, especially important is that the underlying assumptions are completely different in these two cases.

In the second application – the tangency portfolio – all considered asset classes are matched with liabilities, which means that their return is reduced by the return on liabilities. The analysis of the minimum-variance portfolio shows differences between the asset- liability and the asset-only investor in a minimum-variance portfolio context. Anyway the second approach that leads to the tangency portfolio is important as it shows how horizon dynamics affect portfolio choice for a long-term investor and that the allocation differs for the long-term investor who has to cover liability expenses. That explains why hedge funds and commodities are not important in the tangency portfolio. In the tangency portfolio context risk aversion is not modeled. Anyway, it must be expected, that risk-aversion is higher for collective life insurance due to the legal quote. This must be considered when modeling risk aversion. Moreover, the tangency portfolio shows that alternative investments are important for an asset-liability investor. In this example the weight allocated to private equity is about 10% at all considered horizons. This analysis shows that for practical application an investor with liabilities has to include liabilities and horizon effects in the portfolio task, since both affect asset allocation in meaningful ways.

5. Conclusion

This paper analyzes portfolio implications across the investment horizon for an asset-liability investor. Since risk varies across the investment horizon investors should consider time variations in investment opportunities. Based on the model introduced in Campbell and Viceira ([9], [10]) I am able to capture this complex dynamics and furthermore to carve out implications for strategic asset allocation for an asset-liability investor.

I consider the investment problem for a Swiss life insurance company. Simply put, they have to cover liability expenses and therefore are concerned about the matching of assets and liabilities respectively of asset returns and liability expenses. Consequently, this investment problem differs from the classical static optimal portfolio problem. On one hand liabilities are included and on the other hand the planning horizon covers several years.

For the analysis I include the typical asset classes in which a Swiss life insurer invests, which are stocks Switzerland, stocks global, bonds Switzerland, bonds global, hedge funds, private equity, commodities and listed real estate.[18] To work out implications for an asset-liability investor interest rates and returns are modeled with a vector autoregressive model. I use variables that have been identified as return predictors by earlier empirical research, which are in this case credit spread, yield spread and the dividend-price ratio. Moreover and in difference to previous studies I work with nominal returns instead of inflation adjusted returns. I am able to show in my empirical application that expected excess returns and risk shift over time in predictable ways. For this analysis I estimated

18 Only direct real estate investments are not considered as an additional asset class. This because they are due to their strong local nature difficult to compare with the above mentioned asset classes in an analytical and meaningful way.

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the process generating parameters and their evolution over time correspondingly to their historically assessed values. The coefficients are estimated by least squares applied separately to each of the equations.[19] Moreover, to fully focus on risk horizon effects I implied several restrictions. I abstracted from taxation related issues, assumed that there are no transaction costs and ignored that volatility may alter through time. The consideration of these effects may be important in practice as it can be expected that they would alter the outcome. Furthermore I provide unconditional average portfolio allocations for the LHPF respectively for the GMV portfolio and the tangency portfolio - instead of the full range of allocation that would be optimal under different market environments.

To explore the implications on risk across the investment horizon I firstly analyze how volatilities of excess returns for each asset class over liabilities behave across the horizon. And secondly I examine correlations between asset and liability returns. I found that risk for the short-term benchmark asset increases with an increasing investment period. In practice liability investors generally avoid short-term assets since they are not able to cover liability expenses and furthermore are much riskier over the long-term due to roll-forward risk. Furthermore I found mean-reversing effects for stocks, private equity and listed real estate investments. Risk for bonds, hedge funds and commodities is in my empirical application relatively constant across the horizon. However, at this point it must be noted that mean-reversing effects in this study are lower compared with previous comparable studies. This can be attributed to the differences in the data set and macro-economic environment. Moreover, I found that also correlation coefficients change across the investment horizon. Correlations are in this long-term context at least as important as volatilities. These horizon dynamics impact asset allocation. To further evaluate these effects and their impact I apply them in my empirical application and analyze portfolio implications for a Swiss life insurance company. I compare the minimum-variance portfolio for an asset-liability and an asset-only investor. Moreover I analyze the tangency portfolio for an asset-liability investor.

The analysis of the minimum-variance portfolio found that the return predictability introduces a wedge between asset allocation strategies. On one hand for a short- and a long-term investor and on the other hand for an asset-liability and an asset-only investor. The main differences are that the asset-liability investor allocates more to bonds Switzerland and less to the benchmark asset. With an increasing investment horizon this difference increases. Moreover, the asset-liability investor allocates – in this minimum-variance context – more to hedge funds and to listed real estate. This gab further widens for a longer investment horizon. Both investors are short invested in stocks Switzerland, with almost the same weight. For practical implications the analysis of the tangency is more meaningful. In this context each asset return is reduced by liability expenses. The tangency portfolio is the portfolio on the efficient frontier with the maximal Sharpe ratio and has no loading on the risk-free benchmark asset. It can be seen that weight allocated to stocks and bonds varies across the horizon. Moreover, alternative investments, especially private equity, are also important for the asset-liability investor. They have a low correlation with bonds Switzerland and therefore are a good diversifier. Furthermore, correlation with liabilities increases with an increasing horizon, which makes them a good liability hedge. The results obtained show that risk dynamics across the horizon significantly influence the portfolio choice and underline how important it is to include horizon effects – besides the consideration of the liabilities – for optimal asset allocation.

At this point it must be mentioned that it is widely documented in previous academic literature that the possibility of erratic outcomes in portfolio models gets larger the larger the number of assets included. This because of the increasing sensitivity to minor estimation errors in expected returns. I included nine asset classes and liabilities, therefore the danger of error-maximization is large. Anyway, the results are meaningful and underline that dynamics across the horizon should be included for asset allocation tasks. As seen in the tangency portfolio application, a life insurer has to cover high

19 At this point it must be mentioned that "past performance is not a reliable indicator of future performance".

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liability expenses and therefore has to take risk and attempt to earn money at the financial markets. Effectively, the value respectively the required return on assets to cover liabilities depends on many different factors, like discount rates and mortality rates. I replicated liability returns with an average of the interest rate over the last few years. That should be an approximately good indicator. But it would be interesting to see if other liability modeling approaches in such an asset-liability portfolio task would lead to different results.

Moreover, in my empirical application I strove to include all for a Swiss life insurer relevant asset classes. Anyway, I decided to abstract from one asset class – direct real estate investments Switzerland. This because they are due to their local nature and specific characteristics only weakly comparable with other asset classes in a mean-variance set-up. It therefore remains an open question how direct real estate investments could add value for a Swiss life insurer in terms of hedge against liabilities. Furthermore I abstracted from the possibility that companies can hedge against their foreign exchange exposures. As most included asset classes are listed in USD and I decided to translate them into CHF before calculating returns the possibility of hedging would alter the results obtained. The complex interaction between assets, liabilities, long-term investing and hedging may be a starting point for several strands of further research.

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